Difference in difference: Extensions

INFO/STSCI/ILRST 3900: Causal Inference

2 Nov 2023

Logistics

Logistics

- Problem set 5 extended to Sunday Nov 5 at 5pm
- Problem set 6 will be due Nov 16
- Final project writeup due Nov 21 5pm
  - summarize what the authors have done
  - propose a new quantity to estimate
  - 1,500-2,000 words total
- Final project presentations Nov 29 in discussion

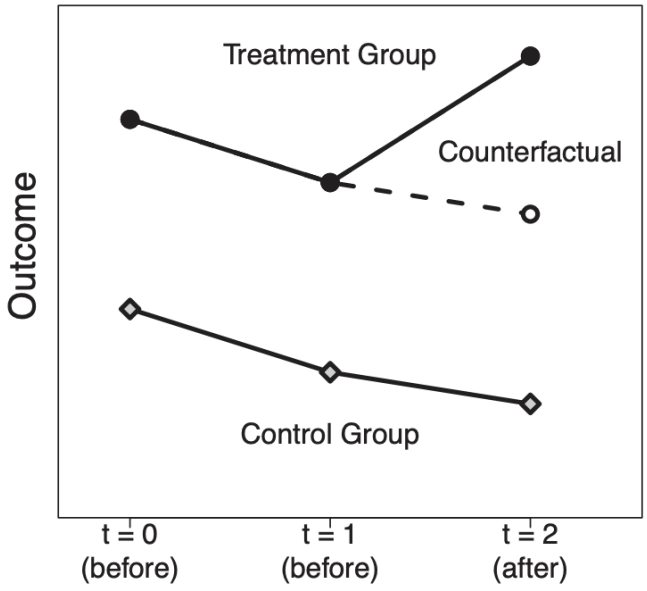
Learning goals for today

At the end of class, you will be able to:

1. Use pre-treatment periods to
   - assess underlying assumptions
   - improve estimation accuracy
   - allow for a more flexible parallel trends assumption
2. and recognize that the parallel assumption remains untestable

Egami, N., & Yamauchi, S. (2023).
Using multiple pretreatment periods to improve difference-in-differences and staggered adoption designs.
Political Analysis, 31(2), 195-212.

Difference in difference



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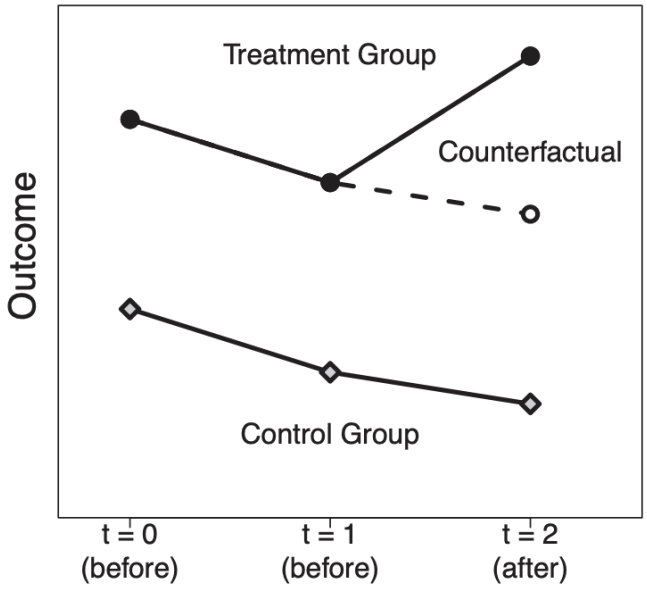
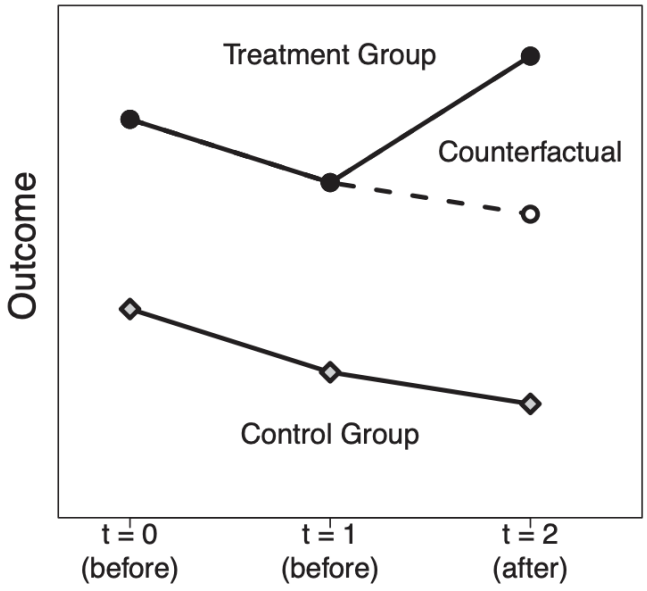


Notation

$$Y^{\text{treatment value}}_{(\text{unit})(\text{time})}$$

Example: Y^0_{i1}
is unit i at time 1
under treatment 0

Difference in difference



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Parallel Trends Assumption (untestable)

$$E(Y^0_{\text{Treated},2} - Y^0_{\text{Treated},1}) = E(Y^0_{\text{Control},2} - Y^0_{\text{Control},1})$$

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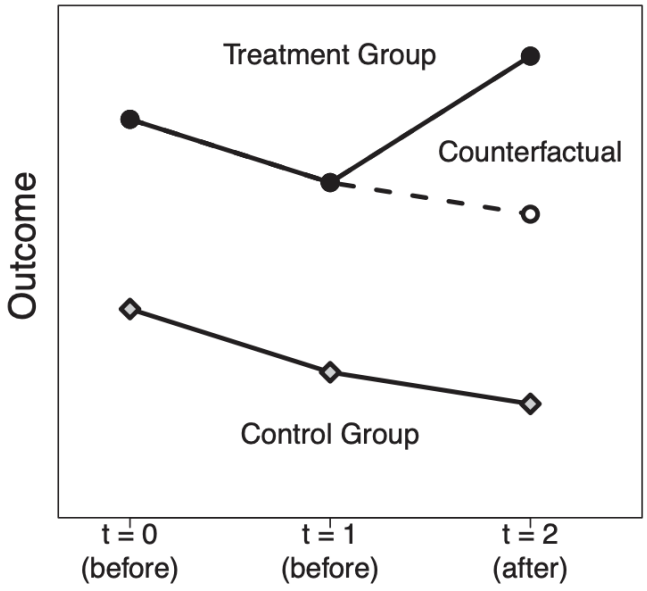

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Extended Parallel Trends (testable)

$$E(Y^0_{\text{Treated},1} - Y^0_{\text{Treated},0}) = E(Y^0_{\text{Control},1} - Y^0_{\text{Control},0})$$

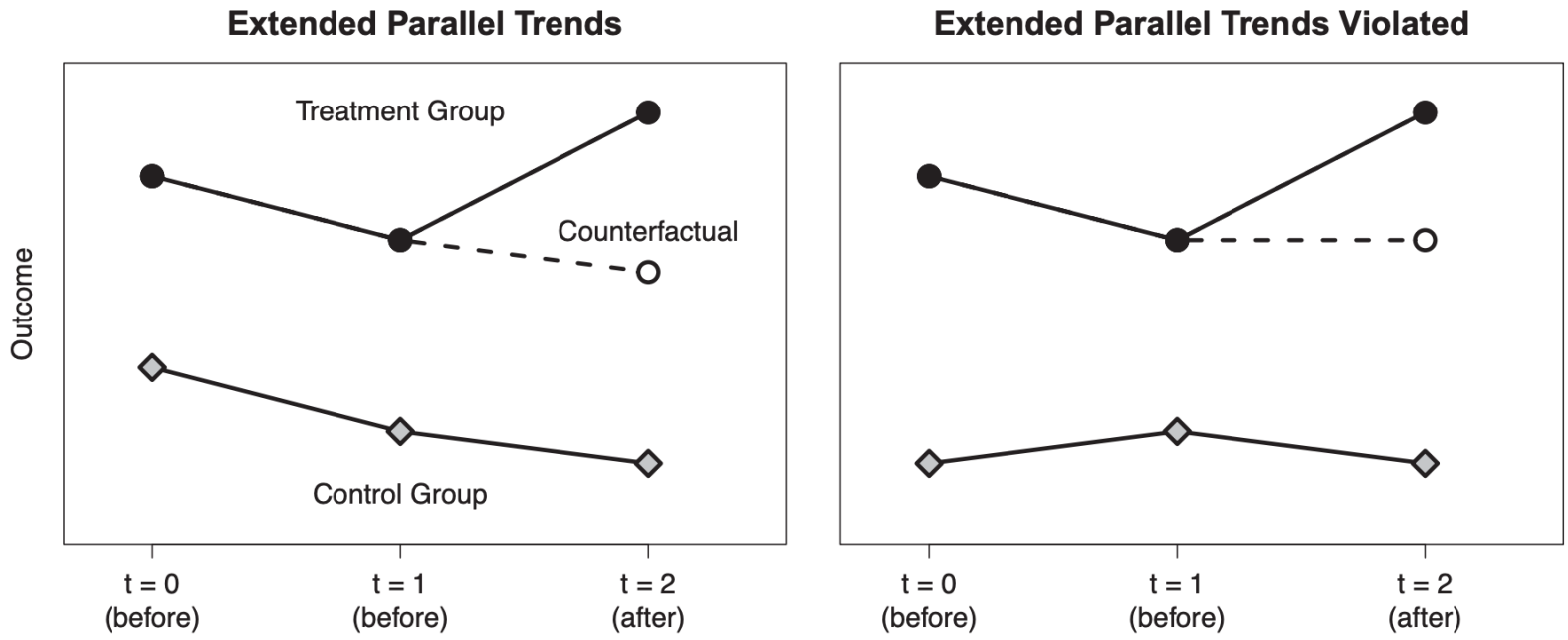
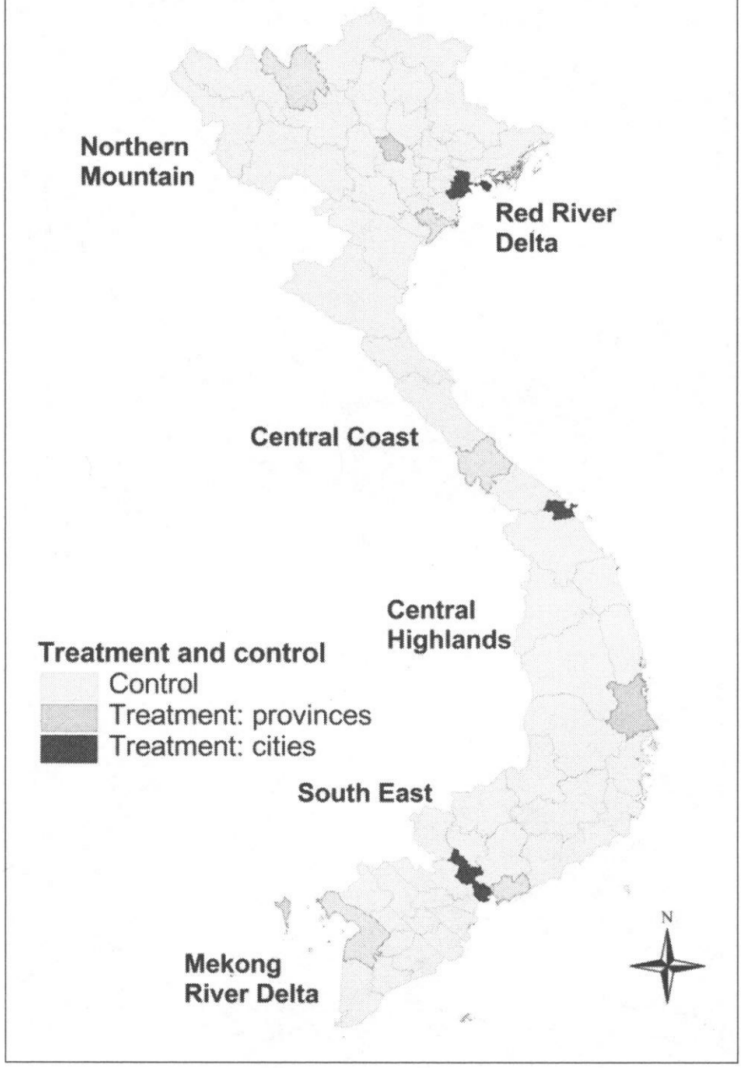
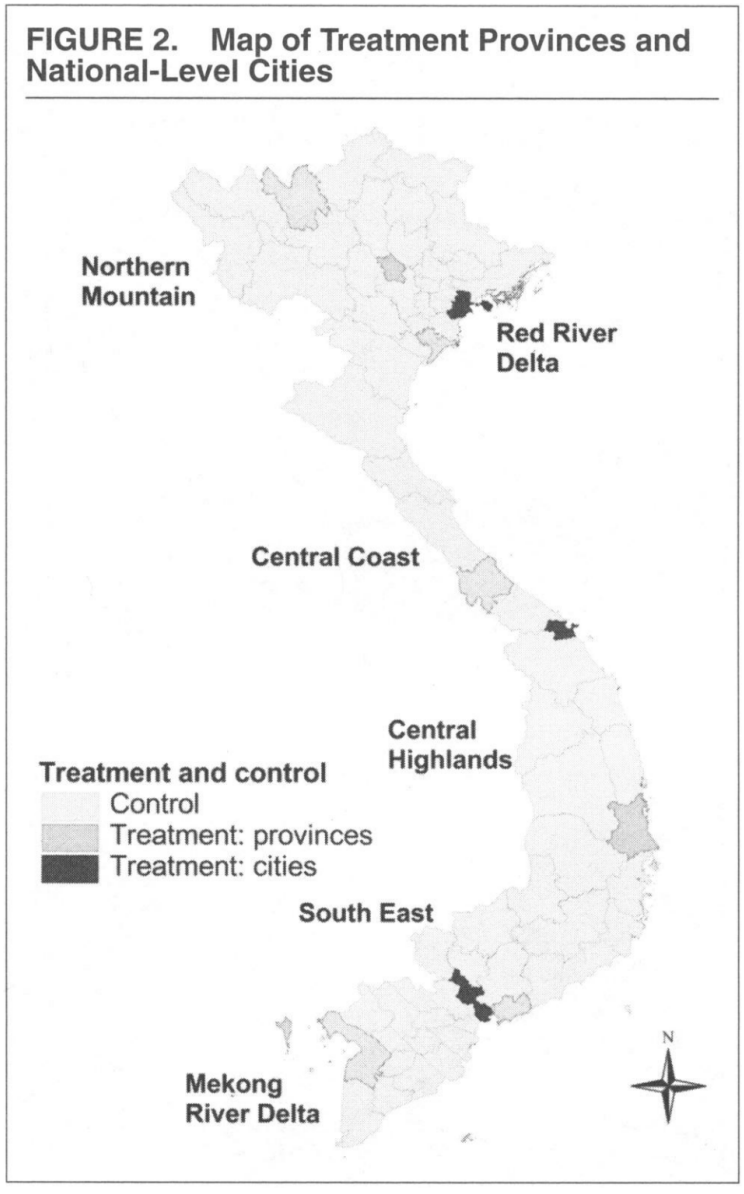

Extended Parallel Trends
Treatment Group
Counterfactual
Control Group
Outcome
t = 0
(before)
t = 1
(before)
t = 2
(after)
Extended Parallel Trends Violated
t = 0
(before)
t = 1
(before)
t = 2
(after)

FIGURE 2. Map of Treatment Provinces and National-Level Cities

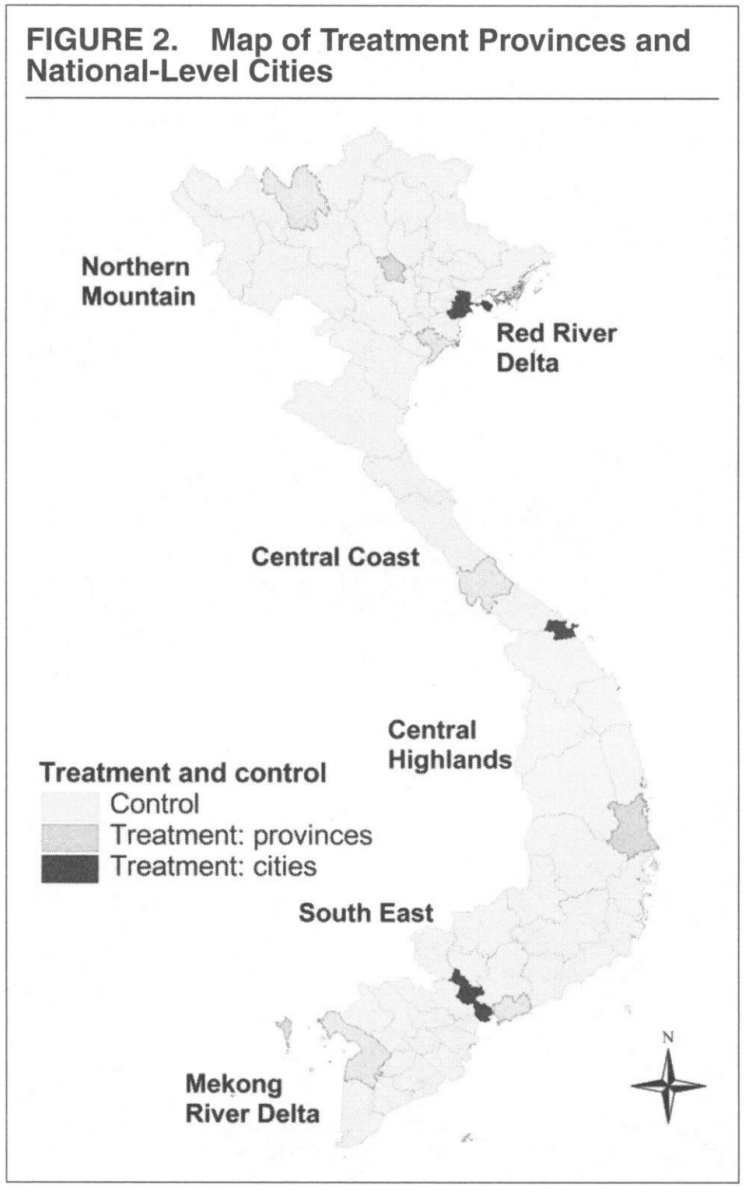




Outcome 1
Education and cultural programs

Is there the following project in the commune?

Investment on culture and education



Outcome 2

Tap water

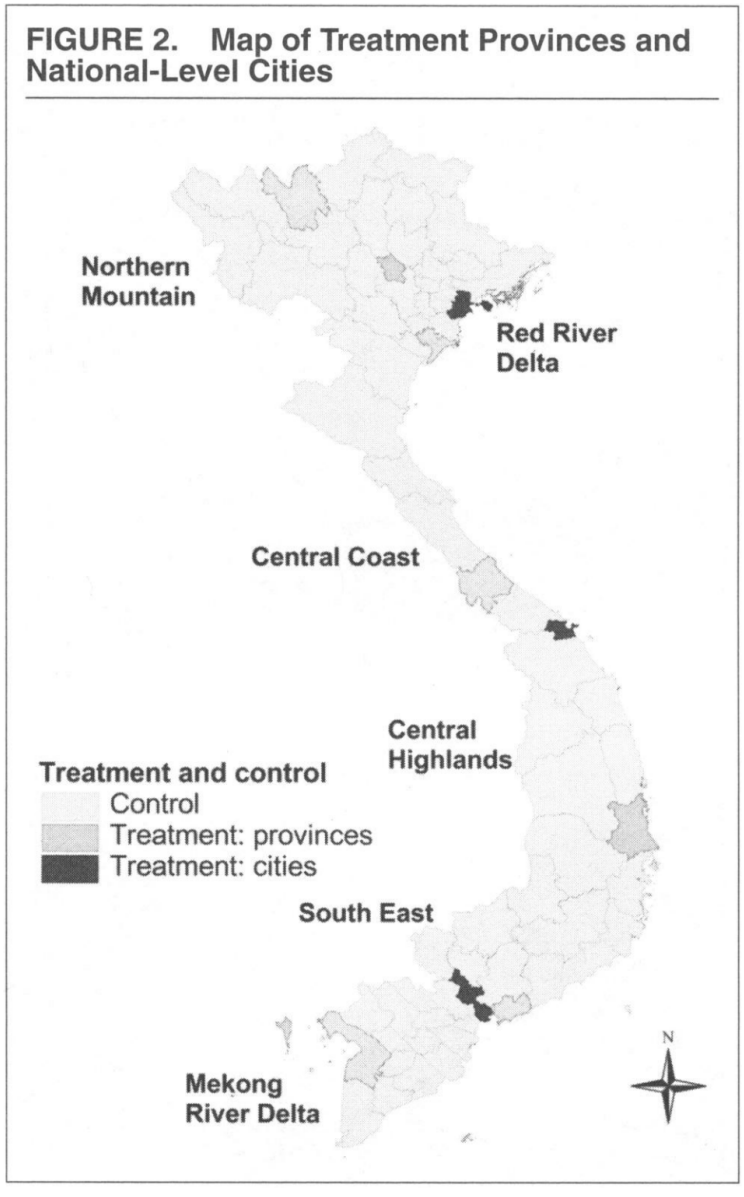
Is there the following project in the commune?

Coded 1

Indoor private piped water
Outdoor private piped water
Public piped water

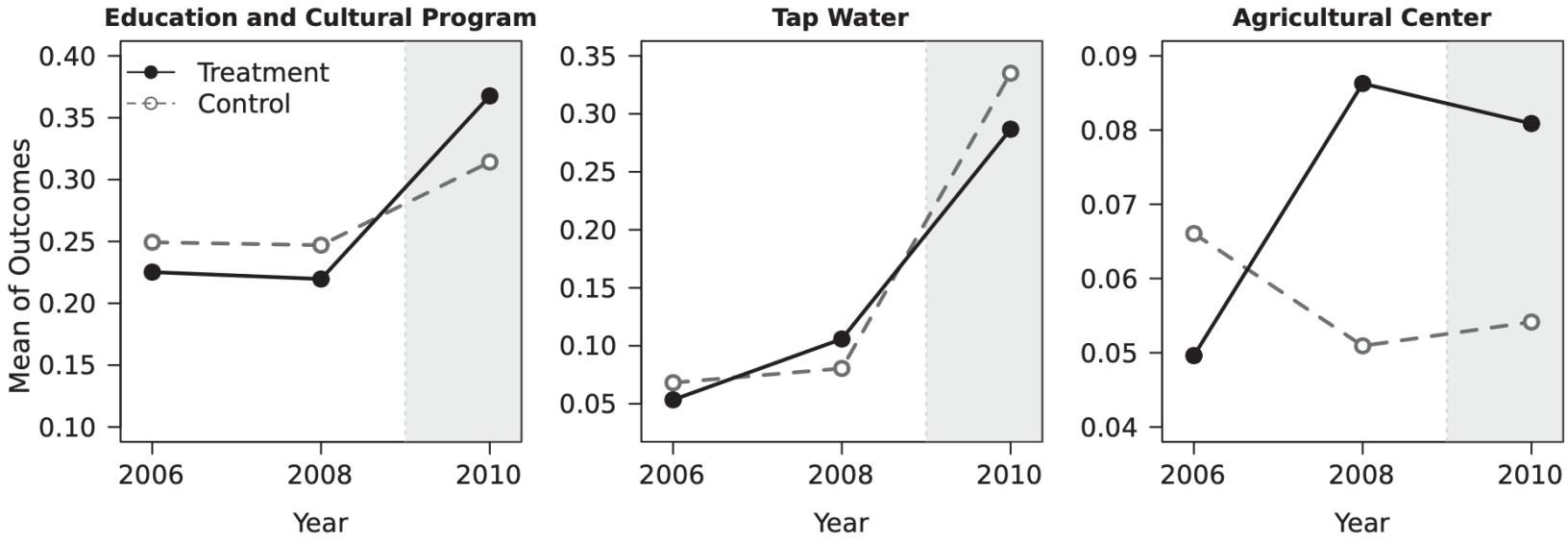
Coded 0

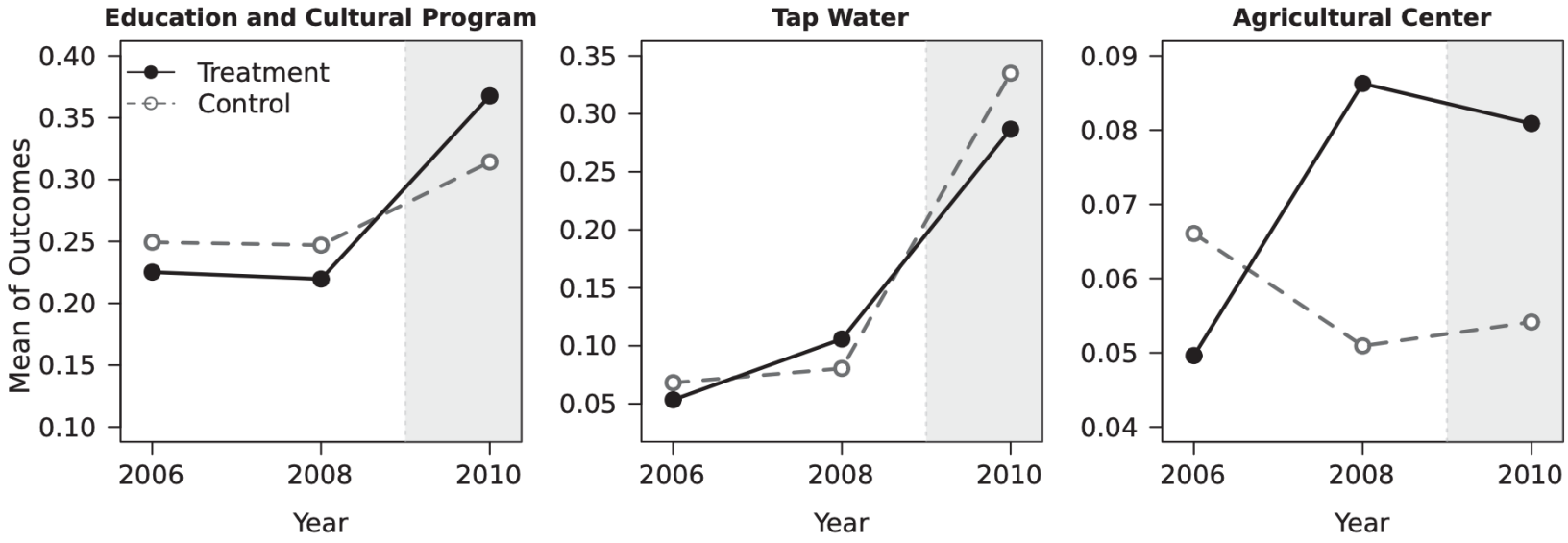
Well water
Well with protection walls
Well without protection walls
Stream water with protection
Stream water without protection
Rainwater
Bottled water
Water brought by pedicab
Tank water
river lake pond



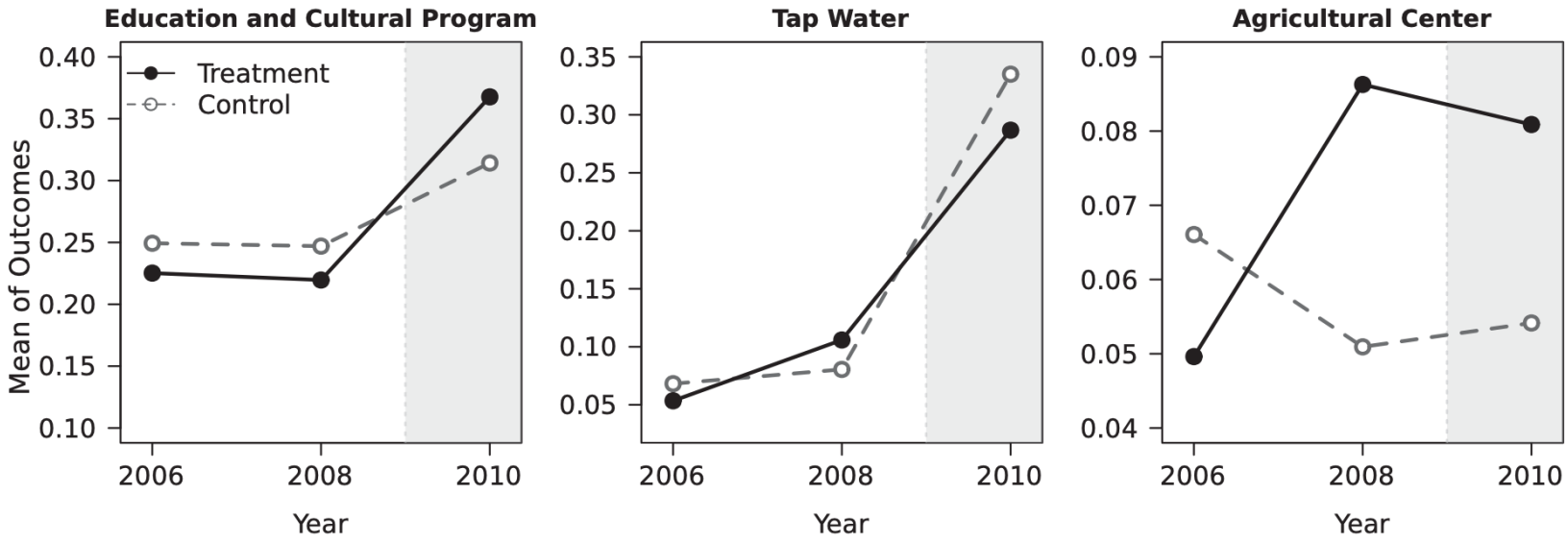
Outcome 3
Agricultural center

Is there any agriculture extension center in this commune?


Education and Cultural Program
Tap Water
Agricultural Center
Treatment
Control
Mean of Outcomes
Year
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.09
0.08
0.07
0.06
0.05
0.04
2006
2008
2010



In each case, do you believe parallel trends?



In each case, do you believe parallel trends?

Table 2. Assessing underlying assumptions using the pretreatment outcomes.

	Estimate	Std. error	*p*-value	95% Std. equivalence CI
Education and cultural program	−0.007	0.096	0.940	[−0.166, 0.166]
Tap water	0.166	0.083	0.045	[−0.302, 0.302]
Agricultural center	0.198	0.082	0.015	[−0.332, 0.332]

Benefit 1: Assessing assumptions

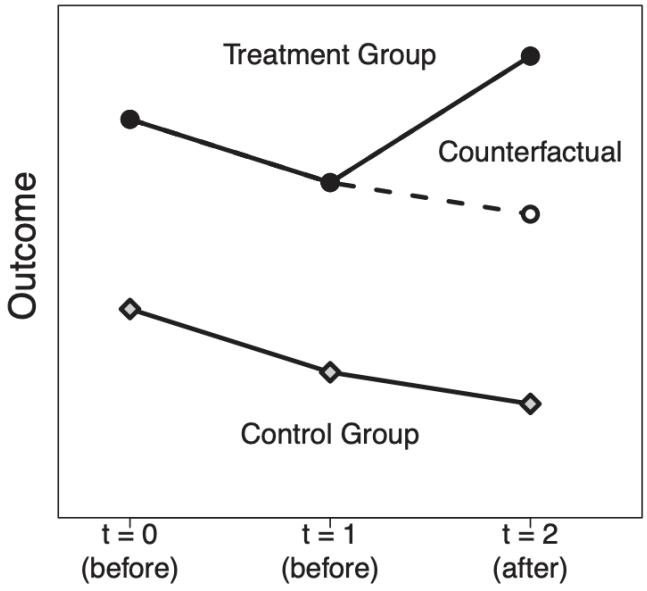
Pre-treatment periods enable us to
assess underlying ssumptions

Parallel trends is untestable, but being parallel
in the pre-treatment period builds confidence

Benefit 2: Improving efficiency

Pre-treatment periods also enable us to
improve estimation accuracy
when parallel trends holds

Benefit 2: Improving efficiency



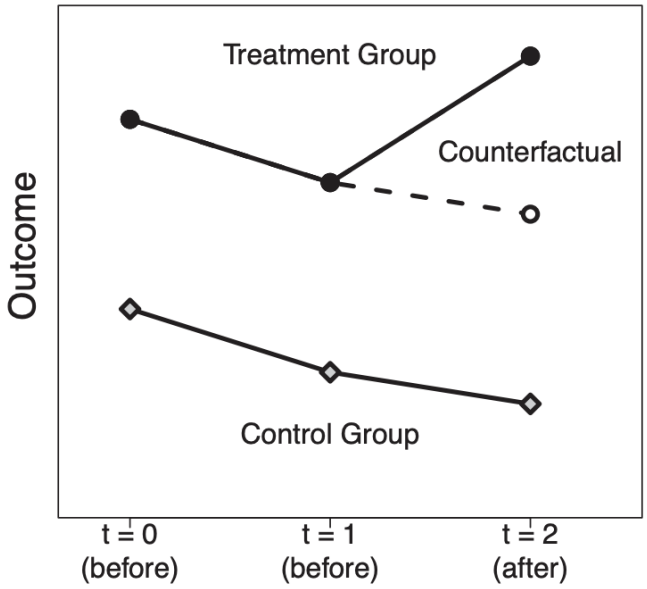
Estimator 1

Estimator 2

Notation

$$Y^{\text{treatment value}}_{\text{(unit)(time)}}$$

Benefit 2: Improving efficiency



Estimator 1

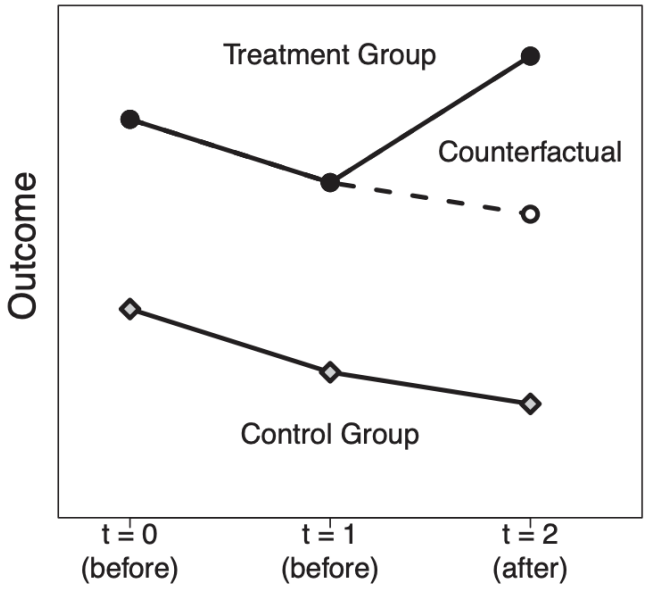
$$\underbrace{(\bar{Y}^1_{T2} - \bar{Y}^0_{T1})}_{\substack{\text{Treatment Group} \\ \text{Time 2 - Time 1}}} - \underbrace{(\bar{Y}^0_{C2} - \bar{Y}^0_{C1})}_{\substack{\text{Control Group} \\ \text{Time 2 - Time 1}}}$$

Estimator 2

Notation

$$Y^{\text{treatment value}}_{(\text{unit})(\text{time})}$$

Benefit 2: Improving efficiency



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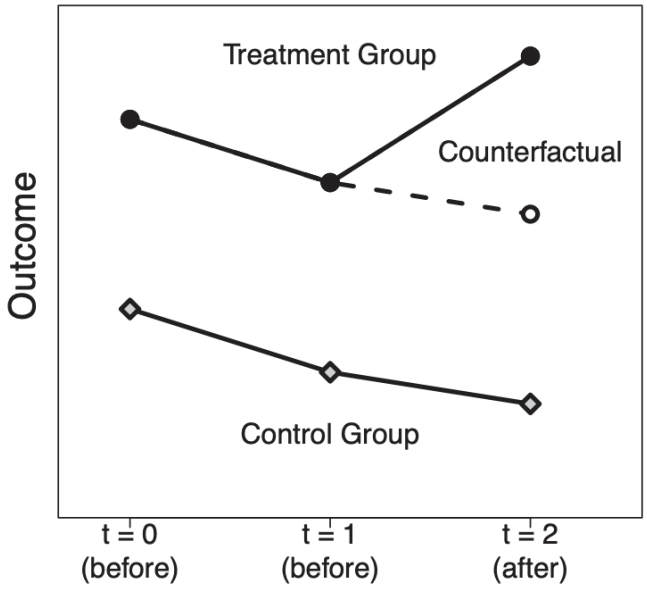
Estimator 2

$$\underbrace{(\bar{Y}^1_{T2} - \bar{Y}^0_{T0})}_{\substack{\text{Treatment Group} \\ \text{Time 2 - Time 0}}} - \underbrace{(\bar{Y}^0_{C2} - \bar{Y}^0_{C0})}_{\substack{\text{Control Group} \\ \text{Time 2 - Time 0}}}$$

Notation

$$Y^{\text{treatment value}}_{(\text{unit})(\text{time})}$$

Benefit 2: Improving efficiency



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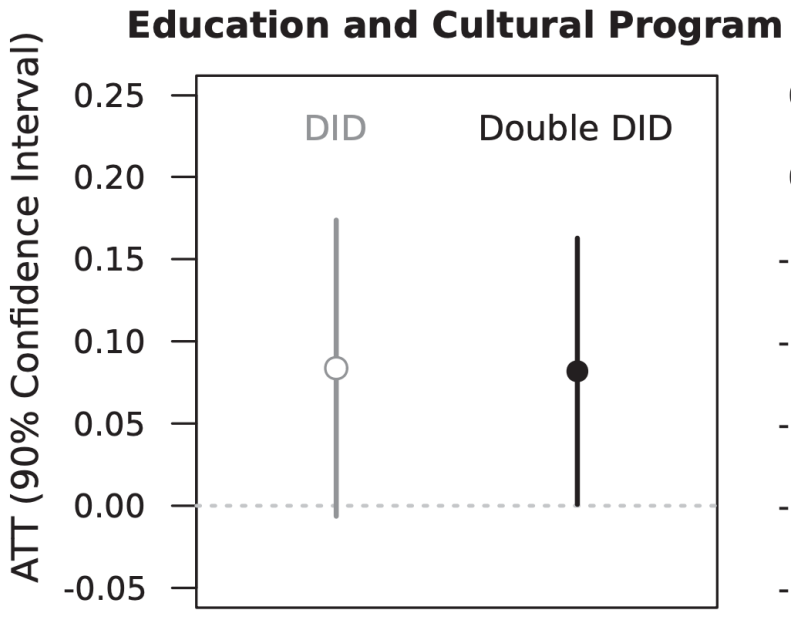
Pooled estimator:
Average the two!

Benefit 2: Improving efficiency



Benefit 3: A more flexible assumption

Pre-treatment periods make it possible to
allow for a more flexible parallel trends assumption

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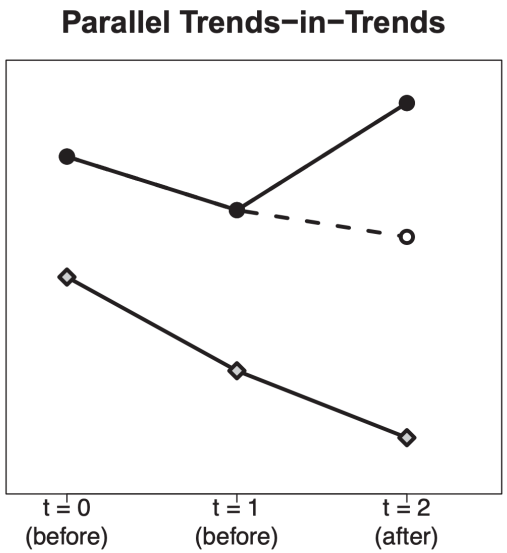

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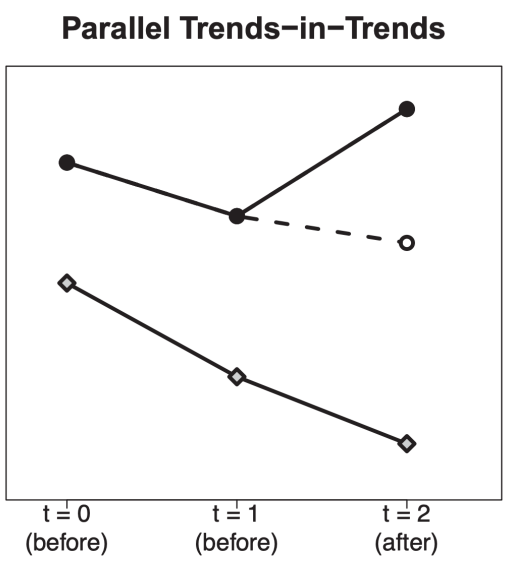

ASSUMPTION 3 (Parallel Trends-in-Trends)

$$\underbrace{\left\{\mathbb{E}[Y_{i2}(0) \mid G_i = 1] - \mathbb{E}[Y_{i1}(0) \mid G_i = 1]\right\}}_{\text{Trend of the treatment group from } t=1 \text{ to } t=2} - \underbrace{\left\{\mathbb{E}[Y_{i1}(0) \mid G_i = 1] - \mathbb{E}[Y_{i0}(0) \mid G_i = 1]\right\}}_{\text{Trend of the treatment group from } t=0 \text{ to } t=1}$$

$$= \underbrace{\left\{\mathbb{E}[Y_{i2}(0) \mid G_i = 0] - \mathbb{E}[Y_{i1}(0) \mid G_i = 0]\right\}}_{\text{Trend of the control group from } t=1 \text{ to } t=2} - \underbrace{\left\{\mathbb{E}[Y_{i1}(0) \mid G_i = 0] - \mathbb{E}[Y_{i0}(0) \mid G_i = 0]\right\}}_{\text{Trend of the control group from } t=0 \text{ to } t=1}.$$

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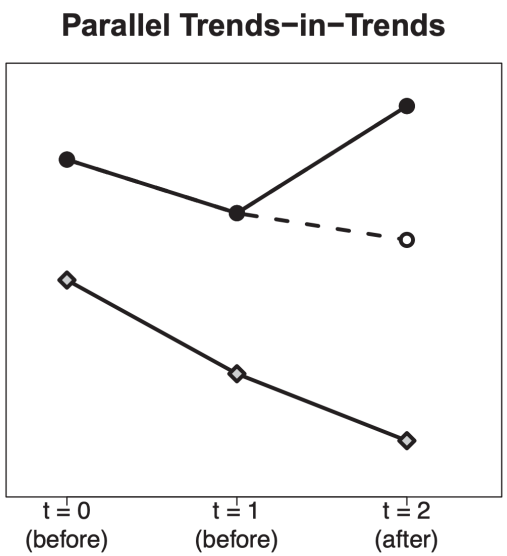

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Sequential DID Estimator

$$\widehat{\tau}_{\texttt{s-DID}} = \left\{\left(\frac{\sum_{i:\, G_i=1} Y_{i2}}{n_{12}} - \frac{\sum_{i:\, G_i=1} Y_{i1}}{n_{11}}\right) - \left(\frac{\sum_{i:\, G_i=0} Y_{i2}}{n_{02}} - \frac{\sum_{i:\, G_i=0} Y_{i1}}{n_{01}}\right)\right\}$$
$$- \left\{\left(\frac{\sum_{i:\, G_i=1} Y_{i1}}{n_{11}} - \frac{\sum_{i:\, G_i=1} Y_{i0}}{n_{10}}\right) - \left(\frac{\sum_{i:\, G_i=0} Y_{i1}}{n_{01}} - \frac{\sum_{i:\, G_i=0} Y_{i0}}{n_{00}}\right)\right\},$$

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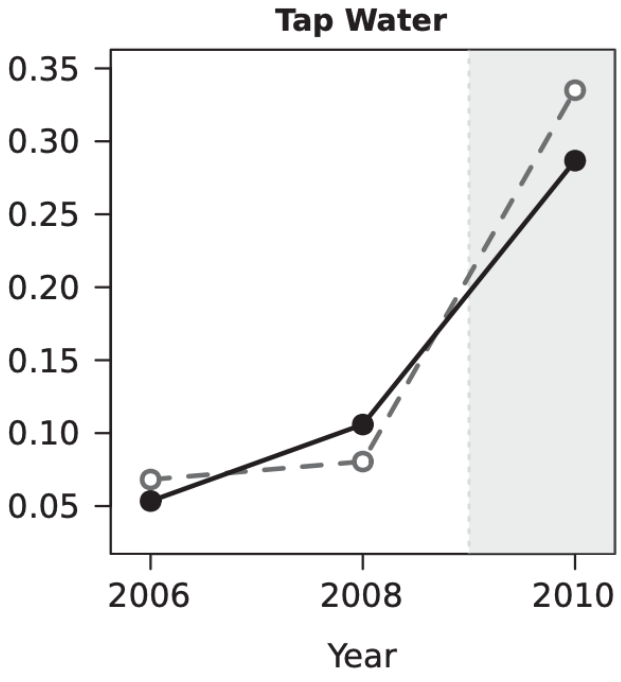

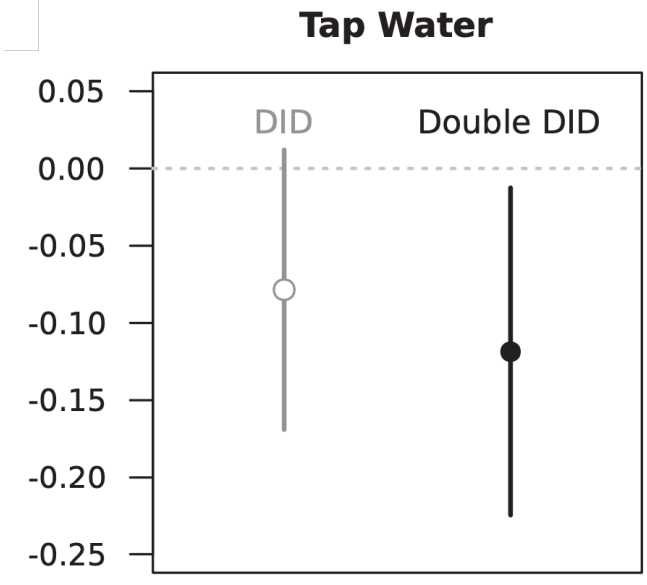

Benefits of multiple pre-treatment periods

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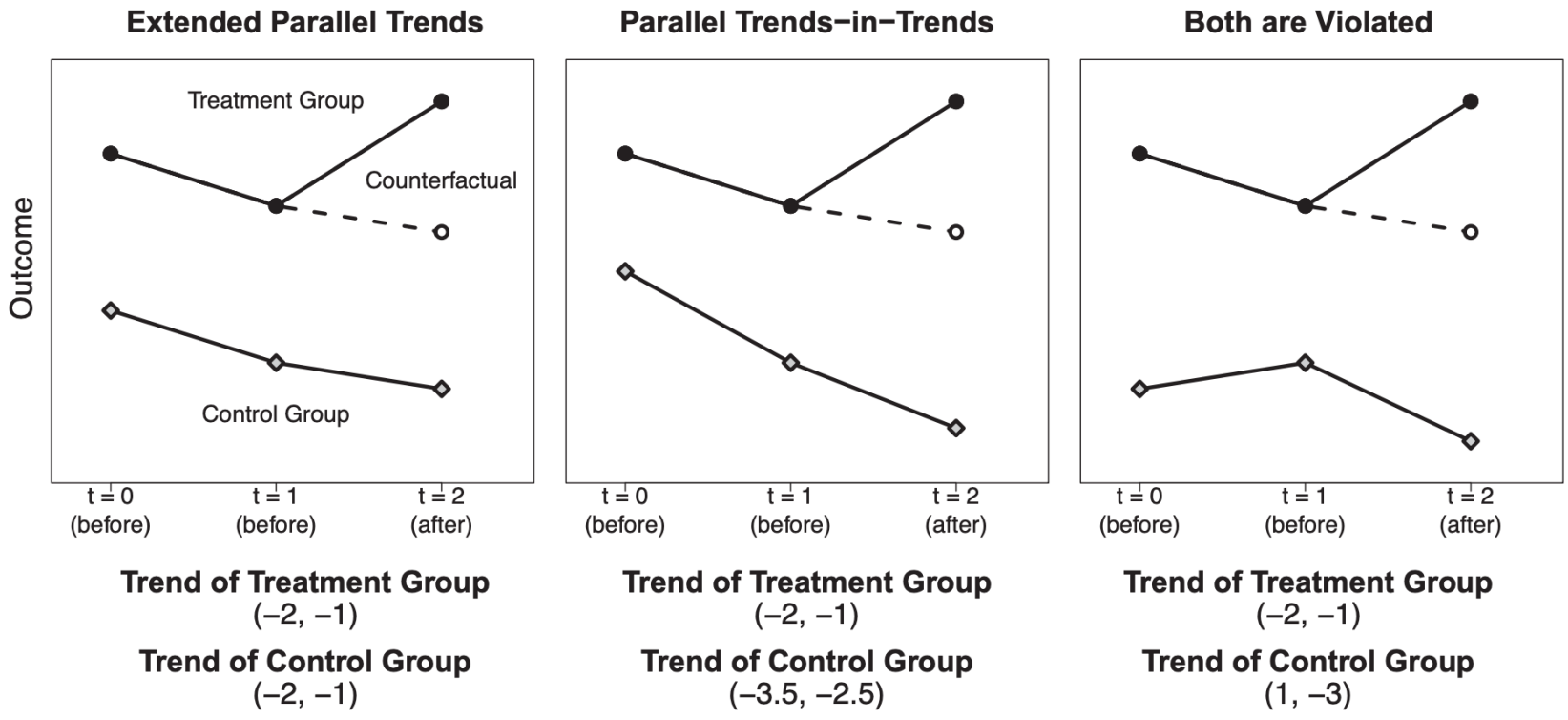

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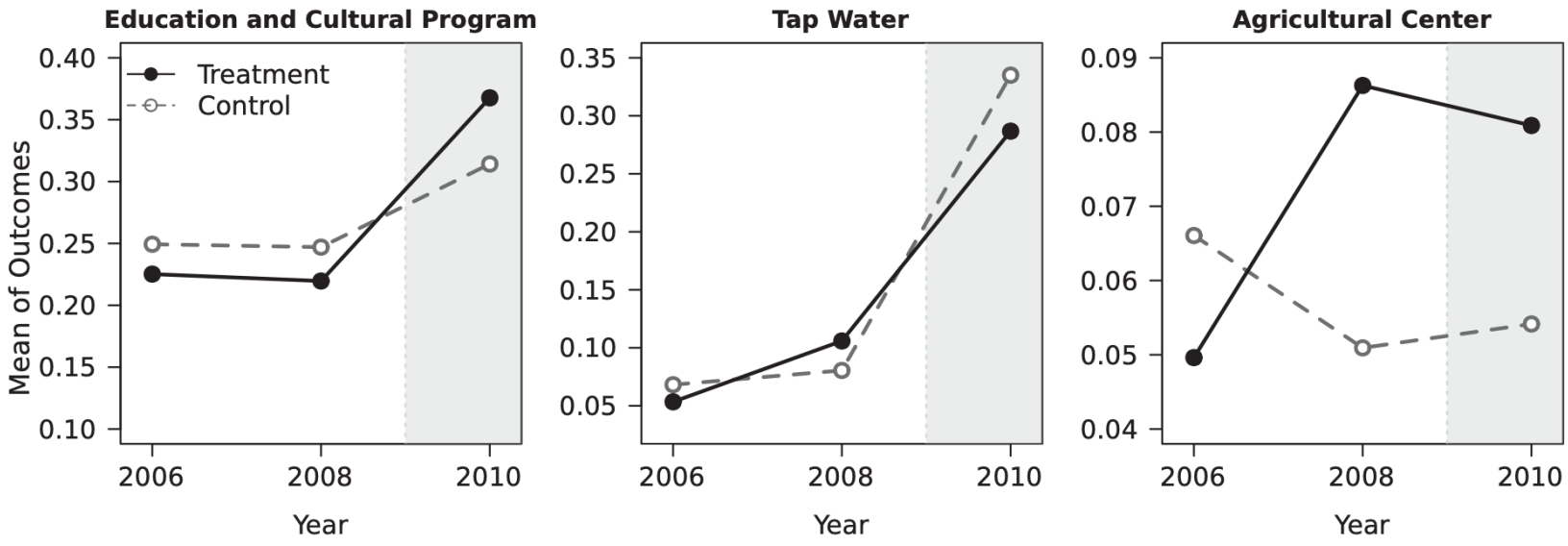

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