Front door identification

INFO/STSCI/ILRST 3900: Causal Inference

12 Oct 2023

Logistics

- Problem Set 4 due Oct 19
- Form for final project groups
  - Writeup due Nov 21
  - Presentations Nov 29

Quick review: Where we are

- define a causal effect
  - treatment, outcome, potential outcomes, target population
- identify a causal effect
  - maps a causal quantity (involving counterfactuals) to a statistical quantity (involving only factual variables)
  - DAGs, conditional exchangeability
- estimate a causal effect
  - statistical modeling, matching, regression

Learning goals for today

At the end of class you will be able to

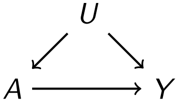
- explain front-door causal identification

More broadly,

1. engage with a new causal identification approach
2. translate that method to code
3. critique the identification assumptions

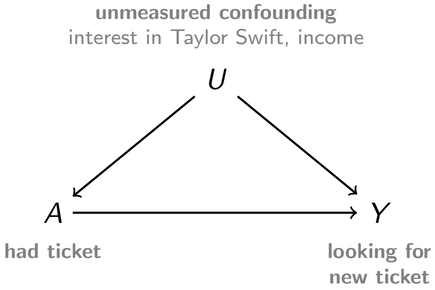
1) Engage with a new causal identification approach

Sometimes a sufficient adjustment set does not exist

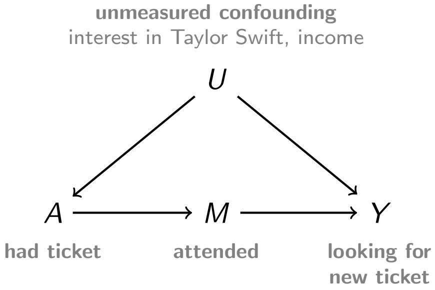


Imagine you are Taylor Swift's head of advertising

Does having a ticket for the Eras Tour increase the probability that a fan look for a future ticket?



As head of advertising, how could you learn about $A \rightarrow Y$?



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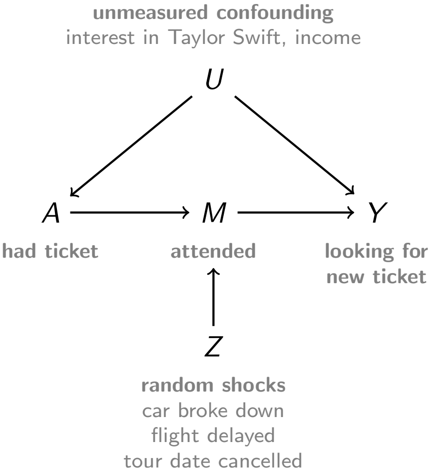
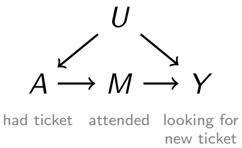
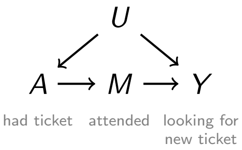


As head of advertising, how could you learn about $A \to Y$?


U
A
M
Y
had ticket
attended
looking for new ticket

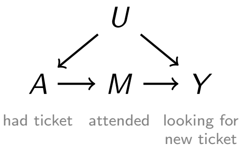
1) $A \rightarrow M$ is identified



1) $A \to M$ is identified

$\mathrm{P}(M^a) =$

for $a = 1$:
would attend
if given a
ticket?

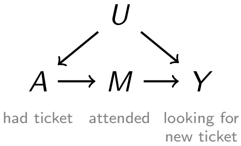


1) $A \to M$ is identified

$$P(M^a) = P(M \mid A = a)$$

for $a = 1$: would attend if given a ticket?

attendance rate among those with tickets



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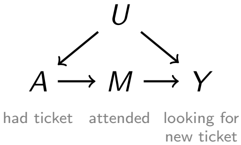

2) $M \to Y$ is identified

1) $A \to M$ is identified

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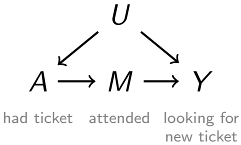

2) $M \to Y$ is identified

$$\mathrm{P}(Y^m)$$

for $m = 1$: would look for new ticket if attended?

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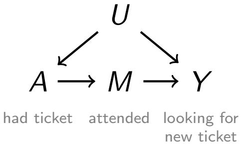

2) $M \to Y$ is identified

$$P(Y^m) = \sum_{a'} P(A = a')P(Y \mid M = m, A = a')$$

for $m = 1$: would look for new ticket if attended?

weighted sum over having ticket

looking for new ticket given attendance?

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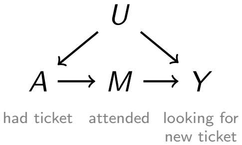

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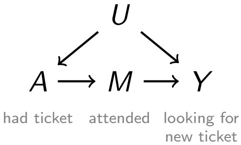
3) $A \to Y$ operates through M

1) $A \to M$ is identified

$$P(M^a) = P(M \mid A = a)$$

for $a = 1$: would attend if given a ticket?

attendance rate among those with tickets



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$$P(Y^m) = \sum_{a'} P(A = a')P(Y \mid M = m, A = a')$$

for $m = 1$: would look for new ticket if attended?

weighted sum over having ticket

looking for new ticket given attendance?

3) $A \to Y$ operates through M

$$P(Y^a) =$$

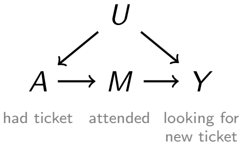
for $a = 1$: would look for future ticket if given ticket?

1) $A \to M$ is identified

$$P(M^a) = P(M \mid A = a)$$

for $a = 1$: would attend if given a ticket?

attendance rate among those with tickets



2) $M \to Y$ is identified

$$P(Y^m) = \sum_{a'} P(A = a')P(Y \mid M = m, A = a')$$

for $m = 1$: would look for new ticket if attended?

weighted sum over having ticket

looking for new ticket given attendance?

3) $A \to Y$ operates through M

$$P(Y^a) = P(Y^{M^a})$$

for $a = 1$: would look for future ticket if given ticket?

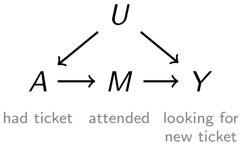
would look for future ticket if attended as if given ticket?

DAG gave us three equations

1) $\mathrm{P}(M^a = m) = \mathrm{P}(M \mid A = a)$

2) $\mathrm{P}(Y^m) = \sum_{a'} \mathrm{P}(A = a')\mathrm{P}(Y \mid M = m, A = a')$

3) $P(Y^a) = \mathrm{P}(Y^{M^a})$



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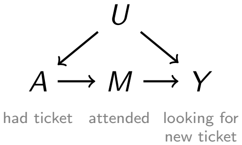
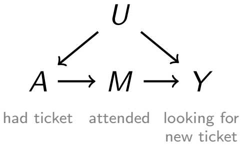


Proof

$$
\begin{aligned}
\mathrm{P}(Y^a) &= P(Y^{M^a}) && \text{by (3)}\\
&= \sum_m \mathrm{P}(M^a = m)\mathrm{P}(Y^m) && \text{law of total prob.}\\
&= \sum_m \mathrm{P}(M = m \mid A = a)\mathrm{P}(Y^m) && \text{by (1)}\\
&= \sum_m \Big(\mathrm{P}(M = m \mid A = a)\\
&\qquad \times \sum_{a'} \mathrm{P}(A = a')\mathrm{P}(Y \mid M = m, A = a')\Big) && \text{by (2)}
\end{aligned}
$$

DAG gave us three equations



1) $\mathrm{P}(M^a = m) = \mathrm{P}(M \mid A = a)$

2) $\mathrm{P}(Y^m) = \sum_{a'} \mathrm{P}(A = a')\mathrm{P}(Y \mid M = m, A = a')$

3) $P(Y^a) = \mathrm{P}(Y^{M^a})$

Result

$$\mathrm{P}(Y^a) = \sum_{m} \mathrm{P}(M = m \mid A = a) \sum_{a'} \mathrm{P}(A = a')\mathrm{P}(Y \mid M = m, A = a')$$

DAG gave us three equations

1) $\mathrm{P}(M^a = m) = \mathrm{P}(M \mid A = a)$

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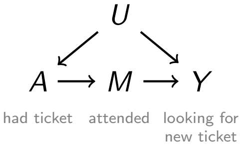


Result

$$\mathrm{P}(Y^a) = \sum_m \mathrm{P}(M = m \mid A = a) \sum_{a'} \mathrm{P}(A = a')\mathrm{P}(Y \mid M = m, A = a')$$

If we intervene to set treatment to the value *a*

then your outcome is a weighted average over the *M* distribution that would result

of the outcome under $M = m$, identified by backdoor adjustment for *A*

2) Translate to code

Translating math to code

$$P(Y^a) = \sum_m P(M = m \mid A = a) \sum_{a'} P(A = a') P(Y \mid M = m, A = a')$$

Translating math to code

$$\mathrm{P}(Y^a) = \sum_m \mathrm{P}(M = m \mid A = a) \sum_{a'} \mathrm{P}(A = a')\mathrm{P}(Y \mid M = m, A = a')$$

Translating math to code

$$P(Y^a) = \sum_m P(M = m \mid A = a) \sum_{a'} P(A = a') P(Y \mid M = m, A = a')$$

```
# Probability of each A
p_A <- data %>%
  # Count size of each group
  group_by(A) %>%
  count() %>%
  # Convert to probability
  ungroup() %>%
  mutate(p_A = n / sum(n)) %>%
  select(A,p_A)
```

Translating math to code

$$\mathrm{P}(Y^a) = \sum_m \mathrm{P}(M = m \mid A = a) \sum_{a'} \mathrm{P}(A = a')\mathrm{P}(Y \mid M = m, A = a')$$

Translating math to code

$$P(Y^a) = \sum_m P(M = m \mid A = a) \sum_{a'} P(A = a') P(Y \mid M = m, A = a')$$

```r
# Probability of Y = 1 given M and A
p_Y_given_M_A <- data %>%
  group_by(A,M) %>%
  summarize(P_Y_given_A_M = mean(Y),
            .groups = "drop")
```

Translating math to code

$$\mathrm{P}(Y^a) = \sum_m \mathrm{P}(M = m \mid A = a) \sum_{a'} \mathrm{P}(A = a')\mathrm{P}(Y \mid M = m, A = a')$$

Translating math to code

$$P(Y^a) = \sum_m P(M = m \mid A = a) \sum_{a'} P(A = a') P(Y \mid M = m, A = a')$$

```
# Probability of Y = 1 under intervention on M
p_Y_under_M <- p_Y_given_M_A %>%
  left_join(p_A, by = "A") %>%
  group_by(M) %>%
  summarize(p_Y_under_M = sum(P_Y_given_A_M  * p_A))
```

Translating math to code

$$\mathrm{P}(Y^a) = \sum_m \mathrm{P}(M = m \mid A = a) \sum_{a'} \mathrm{P}(A = a')\mathrm{P}(Y \mid M = m, A = a')$$

Translating math to code

$$P(Y^a) = \sum_m P(M = m \mid A = a) \sum_{a'} P(A = a')P(Y \mid M = m, A = a')$$

```
# Probability of each M given A
p_M_given_A <- data %>%
  # Count size of each group
  group_by(A, M) %>%
  count() %>%
  # Convert to probability within A
  group_by(A) %>%
  mutate(p_M_under_A = n / sum(n)) %>%
  select(A,M,p_M_under_A)
```

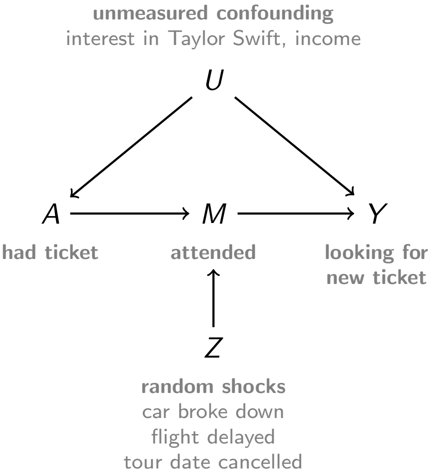
Translating math to code

$$P(Y^a) = \sum_m P(M = m \mid A = a) \sum_{a'} P(A = a') P(Y \mid M = m, A = a')$$

```
# Front door identification
# Probability of Y = 1 under intervention on A
p_Y_under_A <- p_M_given_A %>%
  left_join(p_Y_under_M,
            by = "M") %>%
  group_by(A) %>%
  summarize(estimate = sum(p_M_under_A * p_Y_under_M))
```

Goal 3) Critique the identification assumptions

What edges might need to be added to this DAG?



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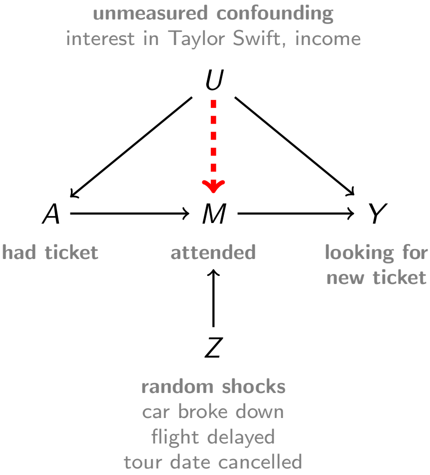

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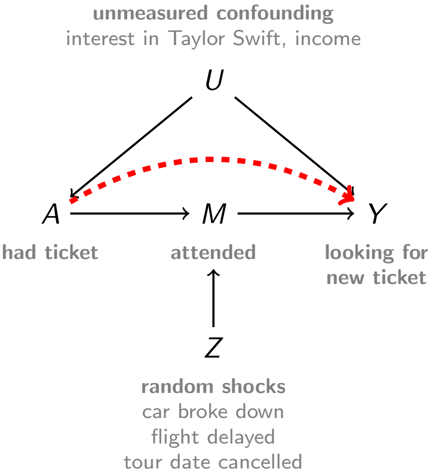

Resources

- Pearl, J. (1995). Causal diagrams for empirical research. Biometrika, 82(4), 669-688.
- Glynn, A. N., & Kashin, K. (2018). Front-door versus back-door adjustment with unmeasured confounding: Bias formulas for front-door and hybrid adjustments with application to a job training program. Journal of the American Statistical Association, 113(523), 1040-1049.

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