# Matching Continued

INFO/STSCI/ILRST 3900: Causal Inference

3 Oct 2023

# Learning goals for today

At the end of class, you will be able to:

1. Understand propensity score matching and coarsened exact matching
2. Use matching methods to estimate causal effects

# Matching: so far

**Goal:** Sample Average Treatment Effect on the Treated

$$E(Y^{a=1} \mid A = 1) - E(Y^{a=0} \mid A = 1)$$

**Potential Solution:** Create a group of untreated individuals, $\mathcal{M}$, which have a **similar distribution of** $L$ to the treated group

$$\frac{1}{n_m} \sum_{i \in \mathcal{M}} Y_i \approx \frac{1}{n_t} \sum_{i : A_i = 1} Y_i^{a=0} \approx E(Y^{a=0} \mid A = 1)$$

**How:**

- Find untreated unit(s) which are similar to each treated unit

# Matching: so far

**Goal:** Sample Average Treatment Effect on the Treated

$$E(Y^{a=1} \mid A = 1) - E(Y^{a=0} \mid A = 1)$$

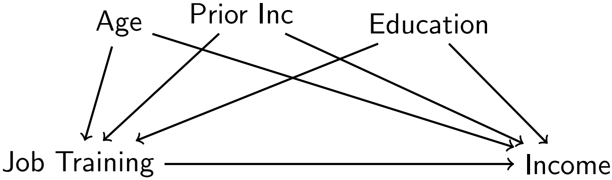
**Potential Solution:** Create a group of untreated individuals, $\mathcal{M}$, which have a **similar distribution of $L$** to the treated group

$$\frac{1}{n_m} \sum_{i \in \mathcal{M}} Y_i \approx \frac{1}{n_t} \sum_{i: A_i = 1} Y_i^{a=0} \approx E(Y^{a=0} \mid A = 1)$$

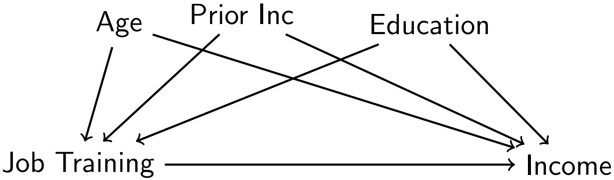
**How:**

- Find untreated unit(s) which are similar to each treated unit
- Define "similar"

# A common distance metric: Propensity scores



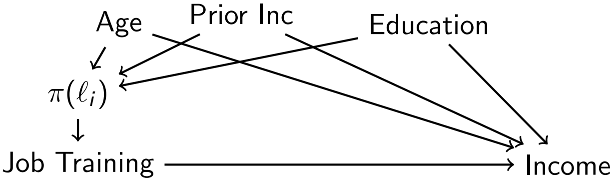
# A common distance metric: Propensity scores




Suppose $\vec{L}$ only affects $A$ through a probability of treatment

$$\pi_i = \pi(\vec{\ell}_i) = P(A_i = 1 \mid \vec{L} = \vec{\ell})$$

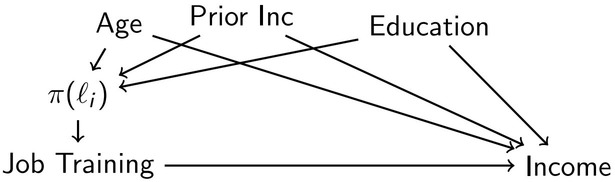
# A common distance metric: Propensity scores




Suppose $\vec{L}$ only affects $A$ through a probability of treatment

$$\pi_i = \pi(\vec{\ell_i}) = P(A_i = 1 \mid \vec{L} = \vec{\ell})$$

# A common distance metric: Propensity scores




Suppose $\vec{L}$ only affects $A$ through a probability of treatment

$$\pi_i = \pi(\vec{\ell_i}) = P(A_i = 1 \mid \vec{L} = \vec{\ell})$$

Conditional exchangeability holds given $\pi(\ell_i)$

# Why propensity scores are nice

# Why propensity scores are nice

- Can match on propensity scores directly instead of $L$

# Why propensity scores are nice

- Can match on propensity scores directly instead of $L$
  - Easy to reason about

# Why propensity scores are nice

- Can match on propensity scores directly instead of $L$
  - Easy to reason about
  - Can directly visualize the univariate matches

# Why propensity scores are nice

- Can match on propensity scores directly instead of $L$
  - Easy to reason about
  - Can directly visualize the univariate matches
- Intuitive: Prioritizes covariates that predict treatment
- Mathematical guarantees on average

# Why propensity scores are nice

- Can match on propensity scores directly instead of $L$
  - Easy to reason about
  - Can directly visualize the univariate matches
- Intuitive: Prioritizes covariates that predict treatment
- Mathematical guarantees on average
  - If our DAG is correct

# Why propensity scores are nice

- Can match on propensity scores directly instead of $L$
  - Easy to reason about
  - Can directly visualize the univariate matches
- Intuitive: Prioritizes covariates that predict treatment
- Mathematical guarantees on average
  - If our DAG is correct
  - If our matches are good

# Why propensity scores are nice

- Can match on propensity scores directly instead of $L$
  - Easy to reason about
  - Can directly visualize the univariate matches
- Intuitive: Prioritizes covariates that predict treatment
- Mathematical guarantees on average
  - If our DAG is correct
  - If our matches are good
  - We should **on average** get a matched group which looks like the the treatment group

$$P(L \mid \pi_i, A_i = 1) = P(L \mid \pi_i, A_i = 0)$$

# A common distance metric: Exact matching

# A common distance metric: Exact matching

- Ideally, we find an exact match for each treated unit

$$d(i,j) = \begin{cases} 0 & \text{if } \vec{L}_i = \vec{L}_j \\ \infty & \text{if } \vec{L}_i \neq \vec{L}_j \end{cases}$$

Often leads to **no matches at all**

# A common distance metric: Coarsened exact matching[1]

[1] Iacus, S. M., King, G., & Porro, G. (2012). Causal inference without balance checking: Coarsened exact matching. Political Analysis, 20(1), 1-24.

# A common distance metric: Coarsened exact matching[1]

- Define $\tilde{\vec{L}}$ to be a coarsened version of $\vec{L}$

[1] Iacus, S. M., King, G., & Porro, G. (2012). Causal inference without balance checking: Coarsened exact matching. Political Analysis, 20(1), 1-24.

# A common distance metric: Coarsened exact matching[1]

- Define $\tilde{\vec{L}}$ to be a coarsened version of $\vec{L}$
  - Example: Age 15-20, 20-25, 25-30, etc

[1] Iacus, S. M., King, G., & Porro, G. (2012). Causal inference without balance checking: Coarsened exact matching. Political Analysis, 20(1), 1-24.

# A common distance metric: Coarsened exact matching[1]

- Define $\vec{\tilde{L}}$ to be a coarsened version of $\vec{L}$
  - Example: Age 15-20, 20-25, 25-30, etc
- Match exactly on $\vec{\tilde{L}}$

[1] Iacus, S. M., King, G., & Porro, G. (2012). Causal inference without balance checking: Coarsened exact matching. Political Analysis, 20(1), 1-24.

# A common distance metric: Coarsened exact matching[1]

- Define $\tilde{\vec{L}}$ to be a coarsened version of $\vec{L}$
  - Example: Age 15-20, 20-25, 25-30, etc
- Match exactly on $\tilde{\vec{L}}$

$$d(i,j) = \begin{cases} 0 & \text{if } \tilde{\vec{L}}_i = \tilde{\vec{L}}_j \\ \infty & \text{if } \tilde{\vec{L}}_i \neq \tilde{\vec{L}}_j \end{cases}$$

[1] Iacus, S. M., King, G., & Porro, G. (2012). Causal inference without balance checking: Coarsened exact matching. Political Analysis, 20(1), 1-24.

# A common distance metric: Coarsened exact matching[1]

- Define $\tilde{\vec{L}}$ to be a coarsened version of $\vec{L}$
  - Example: Age 15-20, 20-25, 25-30, etc
- Match exactly on $\tilde{\vec{L}}$

$$d(i,j) = \begin{cases} 0 & \text{if } \tilde{\vec{L}}_i = \tilde{\vec{L}}_j \\ \infty & \text{if } \tilde{\vec{L}}_i \neq \tilde{\vec{L}}_j \end{cases}$$

- Benefit: Very transparent

[1] Iacus, S. M., King, G., & Porro, G. (2012). Causal inference without balance checking: Coarsened exact matching. Political Analysis, 20(1), 1-24.

# A common distance metric: Coarsened exact matching[1]

- Define $\tilde{\vec{L}}$ to be a coarsened version of $\vec{L}$
  - Example: Age 15-20, 20-25, 25-30, etc
- Match exactly on $\tilde{\vec{L}}$

$$d(i,j) = \begin{cases} 0 & \text{if } \tilde{\vec{L}}_i = \tilde{\vec{L}}_j \\ \infty & \text{if } \tilde{\vec{L}}_i \neq \tilde{\vec{L}}_j \end{cases}$$

- Benefit: Very transparent
- Benefit: Directly targets balance in $L$

[1] Iacus, S. M., King, G., & Porro, G. (2012). Causal inference without balance checking: Coarsened exact matching. Political Analysis, 20(1), 1-24.

# A common distance metric: Coarsened exact matching[1]

- Define $\vec{\tilde{L}}$ to be a coarsened version of $\vec{L}$
  - Example: Age 15-20, 20-25, 25-30, etc
- Match exactly on $\vec{\tilde{L}}$

$$d(i,j) = \begin{cases} 0 & \text{if } \vec{\tilde{L}}_i = \vec{\tilde{L}}_j \\ \infty & \text{if } \vec{\tilde{L}}_i \neq \vec{\tilde{L}}_j \end{cases}$$

- Benefit: Very transparent
- Benefit: Directly targets balance in $L$
- Drawback: May not find a good match for all individuals

[1] Iacus, S. M., King, G., & Porro, G. (2012). Causal inference without balance checking: Coarsened exact matching. Political Analysis, 20(1), 1-24.

# A common distance metric: Coarsened exact matching[1]

- Define $\tilde{\vec{L}}$ to be a coarsened version of $\vec{L}$
  - Example: Age 15-20, 20-25, 25-30, etc
- Match exactly on $\tilde{\vec{L}}$

$$d(i,j) = \begin{cases} 0 & \text{if } \tilde{\vec{L}}_i = \tilde{\vec{L}}_j \\ \infty & \text{if } \tilde{\vec{L}}_i \neq \tilde{\vec{L}}_j \end{cases}$$

- Benefit: Very transparent
- Benefit: Directly targets balance in $L$
- Drawback: May not find a good match for all individuals

[1] Iacus, S. M., King, G., & Porro, G. (2012). Causal inference without balance checking: Coarsened exact matching. Political Analysis, 20(1), 1-24.

# Multivariate distances: Recap

When matching on multivariate $\vec{L}$, you have to define the distance between each pair of confounder values $\vec{\ell}_j$ and $\vec{\ell}_i$

- Manhattan distance
- Euclidean distanace
- Mahalanobis distance
- Coarsened exact distance
- Propensity score distance

There is no right answer! Depends on the setting.

# Multivariate distances: Recap

When matching on multivariate $\vec{L}$, you have to define the distance between each pair of confounder values $\vec{\ell}_j$ and $\vec{\ell}_i$

- Manhattan distance
- Euclidean distanace
- Mahalanobis distance
- Coarsened exact distance
- Propensity score distance

There is no right answer! Depends on the setting.

- Propensity scores are most popular
- Sometimes they are substantively meaningful

# Multivariate distances: Recap

When matching on multivariate $\vec{L}$, you have to define the distance between each pair of confounder values $\vec{\ell}_j$ and $\vec{\ell}_i$

- Manhattan distance
- Euclidean distanace
- Mahalanobis distance
- Coarsened exact distance
- Propensity score distance

There is no right answer! Depends on the setting.

- Propensity scores are most popular
- Sometimes they are substantively meaningful
- Balance only occurs on average

# Evaluate the matched sets

Whatever method, you should check that it worked

- Compare means of $\vec{L}$ (propensity scores) across groups
- Possibly compare interaction cells; e.g., race $\times$ age

# Evaluate the matched sets

Whatever method, you should check that it worked

- Compare means of $\vec{L}$ (propensity scores) across groups
- Possibly compare interaction cells; e.g., race $\times$ age
- Visually assess distribution

# Overlap

- Lack of overlap may indicate violation of positivity assumption

$$P(A = a \mid L = \ell) > 0 \text{ for all a}$$

- Ex: Sarah has no MD training. Would Sarah earn more money if she were a surgeon?

$$P(A = \text{Surgeon} \mid \text{No MD}) = 0$$

- If no good match exists, could be that $P(A = 0 \mid L = \ell) = 0$

# Matching: A word of warning[2]

[2] Sekhon, J. S. (2009). Opiates for the matches: Matching methods for causal inference. Annual Review of Political Science, 12(1), 487-508.

# Matching: A word of warning[2]

$$
\begin{tikzcd}
L \arrow[r] \arrow[rr, bend right] & A \arrow[r] & Y
\end{tikzcd}
$$

# Matching: A word of warning[2]

**Matching works!**

$$
\begin{tikzcd}
L \arrow[r] \arrow[rr, bend right] & A \arrow[r] & Y
\end{tikzcd}
$$

# Matching: A word of warning[2]

**Matching works!**

$$\begin{tikzcd} L \arrow[r] \arrow[rr, bend right] & A \arrow[r] & Y \end{tikzcd}$$

**No help!**

$$\begin{tikzcd} & & U \arrow[dl] \arrow[dr] & \\ L \arrow[r] \arrow[rrr, bend right] & A \arrow[rr] & & Y \end{tikzcd}$$

# Matching: A word of warning[2]

**Matching works!**

$$\begin{tikzcd} L \arrow[r] \arrow[rr, bend right] & A \arrow[r] & Y \end{tikzcd}$$

**No help!**

$$\begin{tikzcd} & & & U \arrow[dl] \arrow[dr] & \\ L \arrow[rr] \arrow[rrrr, bend right] & & A \arrow[rr] & & Y \end{tikzcd}$$

**No help!**

$$\begin{tikzcd} U_1 \arrow[d] \arrow[dr] & & \\ L \arrow[r] \arrow[rr, bend right] & A \arrow[r] & Y \\ U_2 \arrow[u] \arrow[urr, bend right] & & \end{tikzcd}$$

[2] Sekhon, J. S. (2009). Opiates for the matches: Matching methods for causal inference. Annual Review of Political Science, 12(1), 487-508.

# Matching: A word of warning[2]

| **Matching works!** | **No help!** | **No help!** |
|---|---|---|

$$\begin{tikzcd} L \arrow[r] \arrow[rr, bend right] & A \arrow[r] & Y \end{tikzcd}$$

$$\begin{tikzcd} & & U \arrow[dl] \arrow[d] \\ L \arrow[r] \arrow[rr, bend right] & A \arrow[r] & Y \end{tikzcd}$$

$$\begin{tikzcd} U_1 \arrow[d] \arrow[dr] & & \\ L \arrow[r] \arrow[rr, bend right] & A \arrow[r] & Y \\ U_2 \arrow[u] \arrow[urr, bend right] & & \end{tikzcd}$$

Matching is an estimation strategy.

[2] Sekhon, J. S. (2009). Opiates for the matches: Matching methods for causal inference. Annual Review of Political Science, 12(1), 487-508.

# Matching: A word of warning[2]

**Matching works!**

$$\begin{tikzcd} L \arrow[r] \arrow[rr, bend right] & A \arrow[r] & Y \end{tikzcd}$$

**No help!**

$$\begin{tikzcd} & & U \arrow[dl] \arrow[dr] & \\ L \arrow[r] \arrow[rrr, bend right] & A \arrow[rr] & & Y \end{tikzcd}$$

**No help!**

$$\begin{tikzcd} U_1 \arrow[d] \arrow[dr] & & \\ L \arrow[r] \arrow[rr, bend right] & A \arrow[r] & Y \\ U_2 \arrow[u] \arrow[urr, bend right] & & \end{tikzcd}$$

Matching is an estimation strategy.
It does not solve identification problems.

[2] Sekhon, J. S. (2009). Opiates for the matches: Matching methods for causal inference. Annual Review of Political Science, 12(1), 487-508.

# Estimating a causal effect

- If we've matched everything well, we can compare the means

[3] On the statistical role of inexact matching in observational studies. Guo and Rothenhäusler (2023)

# Estimating a causal effect

- If we've matched everything well, we can compare the means
  - Treated group (with a match)
  - Matched control group

[3] On the statistical role of inexact matching in observational studies. Guo and Rothenhäusler (2023)

# Estimating a causal effect

- If we've matched everything well, we can compare the means
  - Treated group (with a match)
  - Matched control group
- We can be extra careful by combining regression + matching
  - If everything is perfect, both should be fine on their own

[3] On the statistical role of inexact matching in observational studies. Guo and Rothenhäusler (2023)

# Estimating a causal effect

- If we've matched everything well, we can compare the means
  - Treated group (with a match)
  - Matched control group
- We can be extra careful by combining regression + matching
  - If everything is perfect, both should be fine on their own
  - Combining can reduce bias
  - Reduces model sensitivity[3]

[3] On the statistical role of inexact matching in observational studies. Guo and Rothenhäusler (2023)

# Code

Let's try this out in R!

# Learning goals for today

At the end of class, you will be able to:

1. Understand propensity score matching and coarsened exact matching
2. Use matching methods to estimate causal effects