Conditional Independence in DAGs

INFO/STSCI/ILRST 3900: Causal Inference

19 Sep 2023

Learning goals for today

At the end of class, you will be able to:

1. Identify whether paths in a causal diagram are open or blocked given a conditioning set
2. Understand why conditioning on colliders differs from conditioning on non-colliders

Logistics

- Ch 6.4 of Hernan and Robins

Causal Graphs

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- Check (marginal) independence by looking at paths in graph

Checking Marginal Independence

$$A \rightarrow Z_1 \rightarrow Z_2 \leftarrow Z_3 \rightarrow Y$$

- Two types of nodes on a path:
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- Two variables are dependant if there is an unblocked path between them

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- Conditional Exchangeability: $Y^a \perp\!\!\!\perp A \mid L$
- How do we tell if a path is open or blocked when conditioning on L?

Open or blocked?

How do we check if a path in the DAG is open or blocked when conditioning on a set of variables L?

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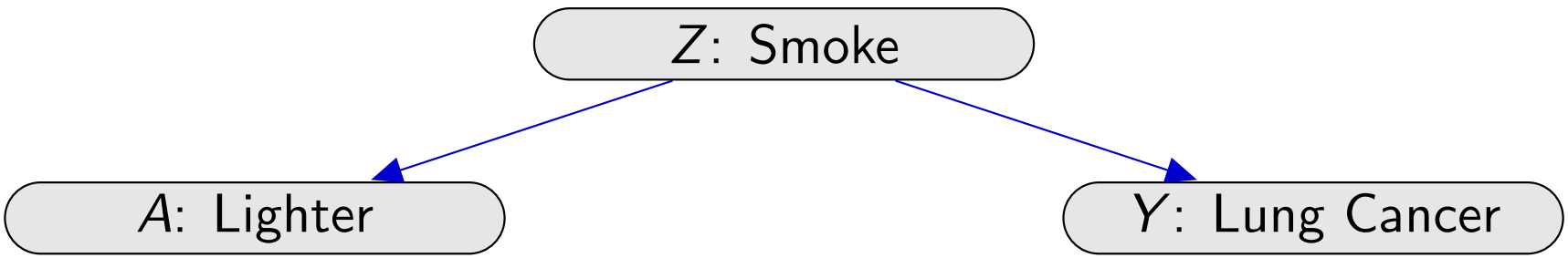
Conditional Exchangeability holds **given** L if all unblocked paths between A and Y are causal paths

Common cause



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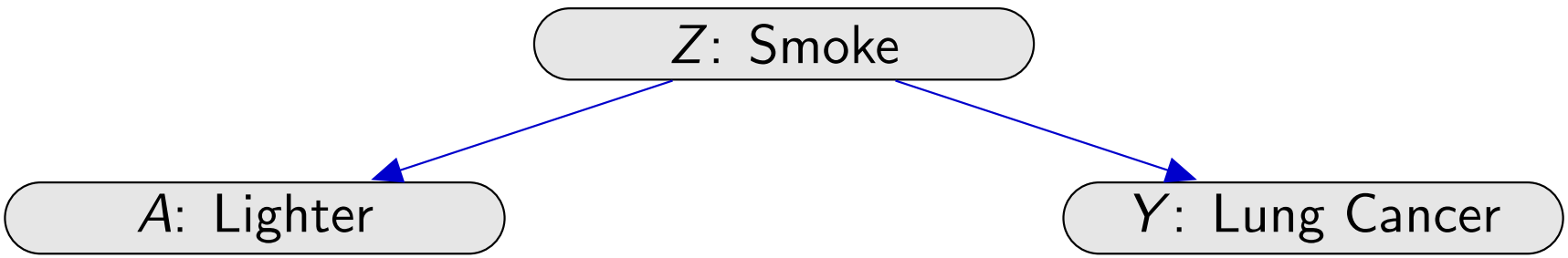

If Z has a causal effect on both A and Y, the path is blocked when we condition on Z

Mediation

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If A effects Y through Z, the path is blocked when we condition on Z

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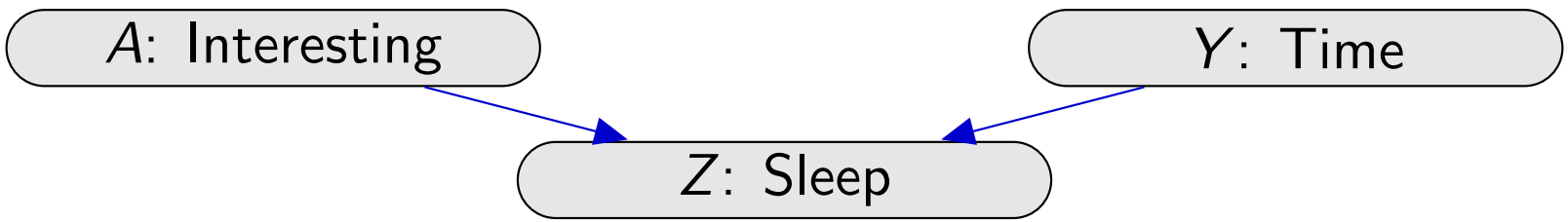

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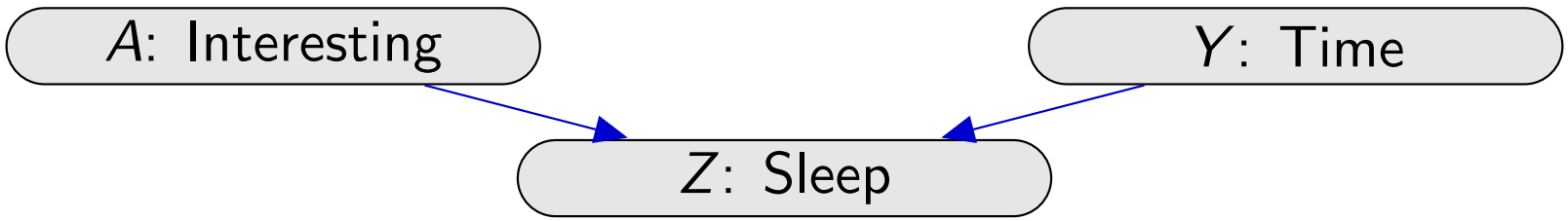
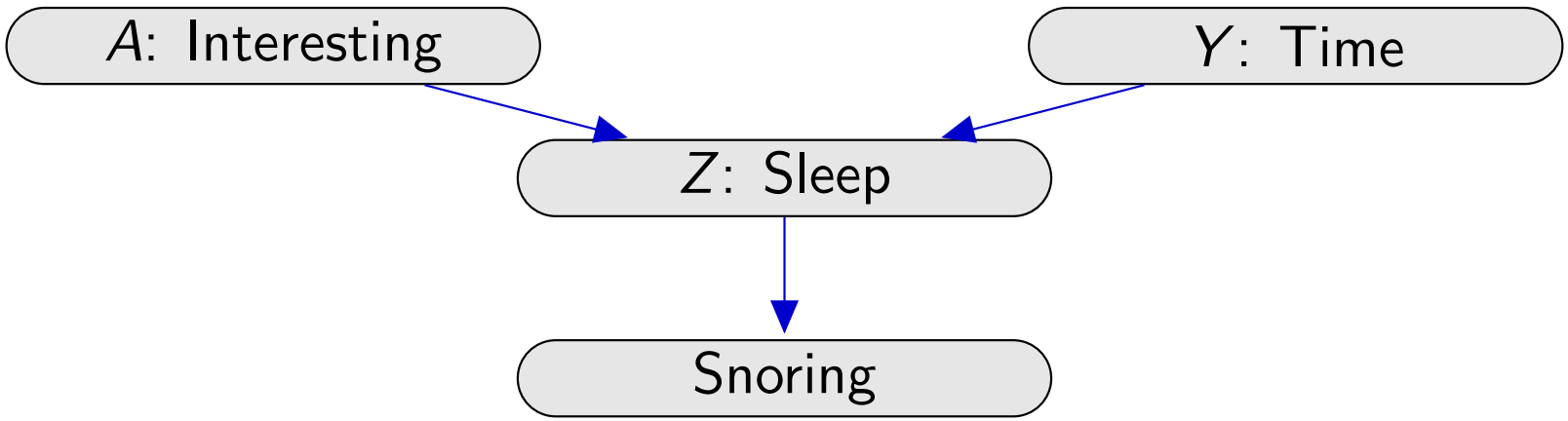


Mathematically,

$$Z = X + Y$$

If we keep Z fixed, but increase X, then to preserve the equation, Y must decrease

Collider



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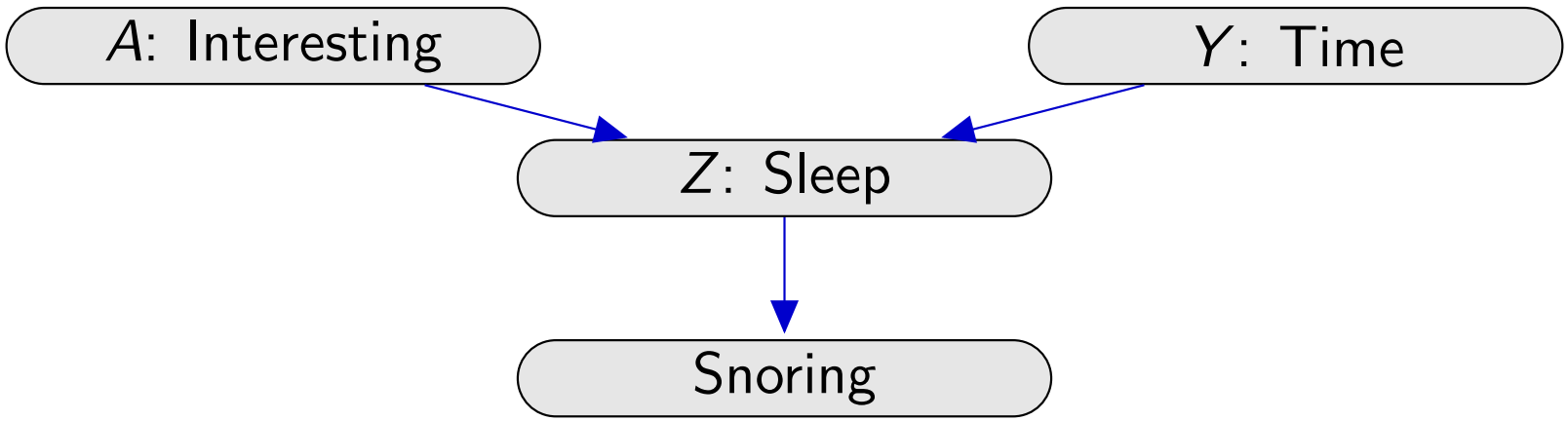

- If there is a causal path $X \to \ldots \to Z$, then Z is a descendant of X

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How to check if a node is open or blocked:

- If non-collider:
  - Open if it is not in the conditioning set
  - Blocked if it is in the conditioning set

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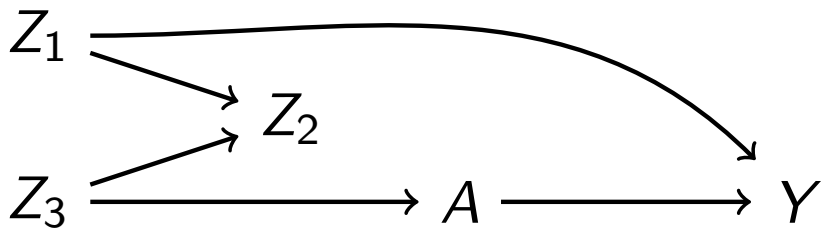
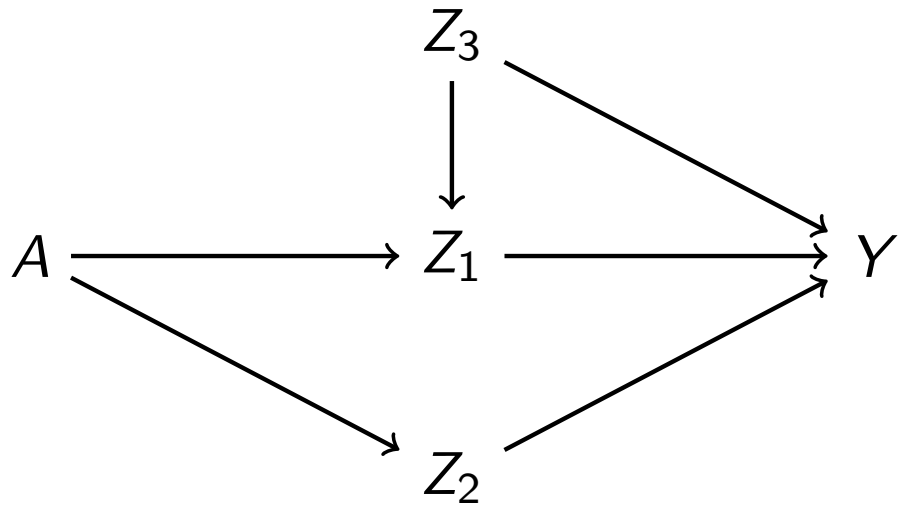


- What are the paths from A to Y?
- When conditioning on $L = \{Z_1\}$ are those paths open or blocked?
- When conditioning on $L = \{Z_2\}$ are those paths open or blocked?
- When conditioning $L = \{Z_1, Z_2\}$ are those paths open or blocked?

Exercise



- What are the paths from A to Y?
- When conditioning on $L = \{Z_2\}$ are those paths open or blocked?
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