Directed Acyclic Graphs: Marginal Independence

Cornell STSCI / INFO / ILRST 3900
Fall 2023
causal3900.github.io

14 Sep 2023

Learning goals for today

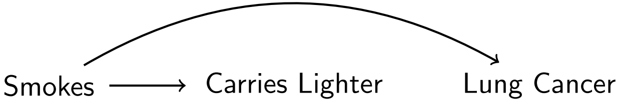
At the end of class, you will be able to

- draw a causal Directed Acyclic Graph
- enumerate edges in the graph
- read statistical dependence of nodes in the graph
- determine marginal exchangeability in the graph

After class:

- Hernán and Robins 2020 Chapter 6.1 and 6.2

Directed Acyclic Graphs (DAGs) formalize **causal beliefs**



Causal beliefs:

1) Smoking may cause you to carry a lighter
2) Smoking may cause lung cancer
3) Carrying a lighter does not cause lung cancer

Directed Acyclic Graphs (DAGs) formalize **causal beliefs**

$$\begin{tikzcd}
\text{Smokes} \arrow[r] \arrow[rr, bend left] & \text{Carries Lighter} & \text{Lung Cancer}
\end{tikzcd}$$

Directed Acyclic Graphs (DAGs) formalize **causal beliefs**

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Nodes represent random variables

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Nodes represent random variables

Edges represent direct causal effects

Directed Acyclic Graphs (DAGs) formalize **causal beliefs**

Nodes represent random variables

Edges represent direct causal effects

Requirement	Any node with edges to at least two other nodes must be included

Directed Acyclic Graphs (DAGs) formalize **statistical dependence**

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Directed Acyclic Graphs (DAGs) formalize **statistical dependence**

$$\begin{tikzcd} \text{Smokes} \arrow[r] \arrow[rr, bend left] & \text{Carries Lighter} & \text{Lung Cancer} \end{tikzcd}$$

(Smokes, Carries Lighter) are statistically dependent

— because (Smokes) causes (Carries Lighter)

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— because (Smokes) causes (Carries Lighter)

(Smokes, Lung Cancer) are statistically dependent
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(Carries Lighter, Lung Cancer) are statistically dependent
— because (Smokes) causes both

Task. Propose a rule for when two nodes are dependent

Two nodes are dependent if and only if __________

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Possible rule (not yet correct)	Two nodes are dependent if and only if they are connected by a path

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Possible rule (not yet correct) — Two nodes are dependent if and only if they are connected by a path

Path — A sequence of edges connecting two nodes

Smokes $\rightarrow$ Carries Lighter
Smokes $\rightarrow$ Lung Cancer
Carries Lighter $\leftarrow$ Smokes $\rightarrow$ Lung Cancer

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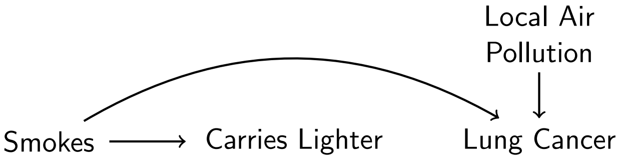

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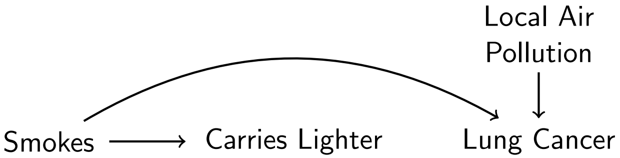

(Local Air Pollution) causes (Lung Cancer)

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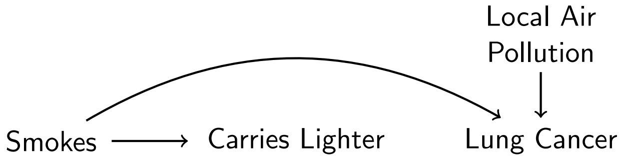

(Local Air Pollution) causes (Lung Cancer)

There are no common causes of (Smokes, Local Air Pollution)

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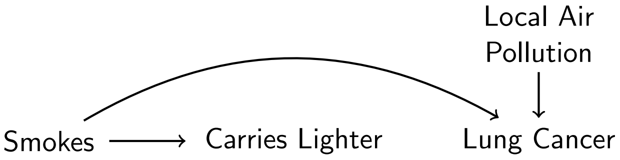

(Local Air Pollution) causes (Lung Cancer)

There are no common causes of (Smokes, Local Air Pollution)

There is a path: (Smokes) $\rightarrow$ (Lung Cancer) $\leftarrow$ (Local Air Pollution)

Task. Propose a rule for when two nodes are dependent

$$
\begin{tikzcd}
 & & \text{Local Air Pollution} \arrow[d] \\
\text{Smokes} \arrow[r] \arrow[rr, bend left] & \text{Carries Lighter} & \text{Lung Cancer}
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$$

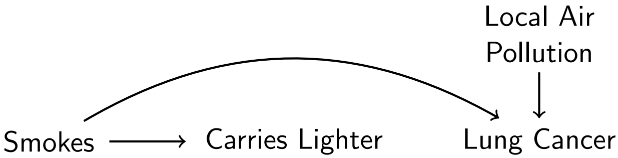
(Local Air Pollution) causes (Lung Cancer)

There are no common causes of (Smokes, Local Air Pollution)

There is a path: (Smokes) $\rightarrow$ (Lung Cancer) $\leftarrow$ (Local Air Pollution)

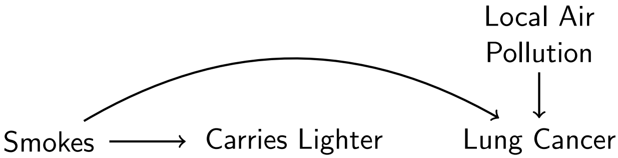
Is (Smokes) statistically related to (Local Air Pollution)?

Task. Propose a rule for when two nodes are dependent



Lung cancer is a **collider** on the path
(Smokes) $\rightarrow$ (Lung Cancer) $\leftarrow$ (Local Air Pollution)

Task. Propose a rule for when two nodes are dependent



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Collider A node on a path where two edges collide $\rightarrow \bullet \leftarrow$

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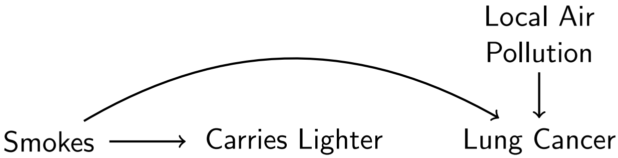

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A collider **blocks the path**.
A blocked path does not create statistical dependence.

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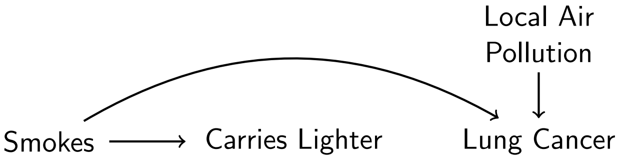

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A blocked path does not create statistical dependence.

Intuition: If two variables affect one outcome,
that does not make those two variables related

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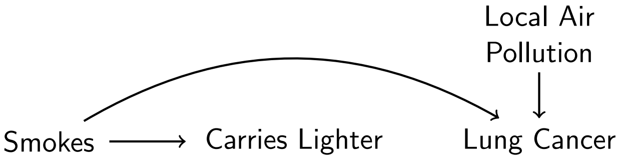

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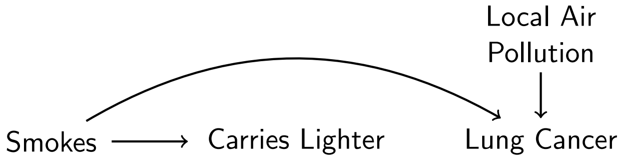
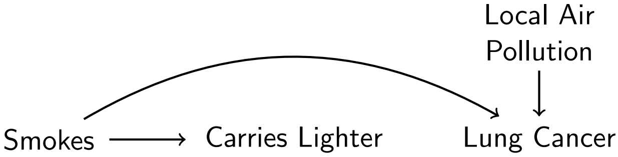


Possible rule (not yet correct)

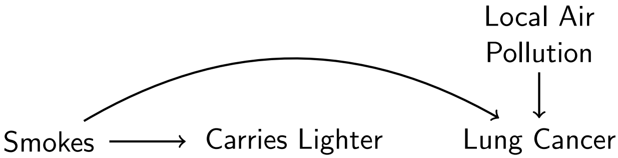
Two nodes are dependent if and only if they are connected by a path

Task. Propose a rule for when two nodes are dependent



Possible rule (not yet correct) Two nodes are dependent if and only if they are connected by an unblocked path

Task. Propose a rule for when two nodes are dependent



Rule Two nodes are dependent if and only if they are connected by an unblocked path

DAGs help us reason about exchangeability

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DAGs tell us why two variables are statistically dependent

- A set of unblocked paths

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Exchangeability requires statistical independence: $A \perp\!\!\!\perp Y^a$

- Exchangeability holds if the only reason A and Y are related is the causal effect of A on Y

DAGs help us reason about exchangeability

DAGs tell us why two variables are statistically dependent

- A set of unblocked paths

Exchangeability requires statistical independence: $A \perp\!\!\!\perp Y^a$

- Exchangeability holds if the only reason A and Y are related is the causal effect of A on Y

Exchangeability holds if all unblocked paths between A and Y are causal paths that point from A to Y

DAGs help us reason about exchangeability

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Procedure

1) List all paths between A to Y
2) Cross out the blocked paths
3) Exchangeability holds if all remaining paths are causal

DAGs help us reason about exchangeability

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1) List all paths between A to Y
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DAG 1

$$\begin{tikzcd} & U \arrow[d] \\ A \arrow[r] & Y \end{tikzcd}$$

DAG 2

$$\begin{tikzcd} U \arrow[d] \arrow[dr] & \\ A \arrow[r] & Y \end{tikzcd}$$

DAG 3

$$\begin{tikzcd} & U \arrow[dr] & \\ A \arrow[ur] \arrow[rr] & & Y \end{tikzcd}$$

DAG 4

$$\begin{tikzcd} & U \\ A \arrow[ur] \arrow[r] & Y \arrow[u] \end{tikzcd}$$

DAG 5

$$\begin{tikzcd} U_1 \arrow[dd] \arrow[dr] & & U_2 \arrow[dl] \arrow[dd] \\ & U_3 & \\ A \arrow[rr] & & Y \end{tikzcd}$$

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