

Measuring of Causal effects and Standardization

INFO/STSCI/ILRST 3900: Causal Inference

5 Sep 2023

Learning goals for today

At the end of class, you will be able to:

1. Describe different ways to quantitatively measure a causal effect
2. Estimate the average causal effect using data from a conditionally randomized experiment

Logistics

- ▶ Ch 1.3 and 2.3 in Hernan and Robins 2023
- ▶ Problem Set 1 due last Thursday

Conditional randomization

Exchangeability may not hold in every randomized experiment

- ▶ Age ≥ 55 receive vaccine with $2/3$; more likely to get COVID if treated
- ▶ Age < 55 get vaccine with probability $1/2$; less likely to get COVID if treated

Conditional randomization

Exchangeability may not hold in every randomized experiment

- ▶ Age ≥ 55 receive vaccine with $2/3$; more likely to get COVID if treated
- ▶ Age < 55 get vaccine with probability $1/2$; less likely to get COVID if treated
- ▶ Exchangeability does not hold in entire population
- ▶ Exchangeability holds within each sub-population
- ▶ Two separate experiments; both are exchangeable

Conditional randomization

- ▶ **Marginal exchangeability:** $Y^a \perp\!\!\!\perp A$ for all a
- ▶ **Conditional exchangeability:** $Y^a \perp\!\!\!\perp A \mid L$ for all a
The potential outcomes are independent of treatment **conditional on L**

Conditional randomization

- **Stratification:** We can directly estimate causal effect within each sub-population (or stratum)

$$\begin{aligned} E(Y | A = a, L = \ell) &\stackrel{\text{consis}}{=} E(Y^a | A = a, L = \ell) \\ &\stackrel{\text{exchange}}{=} E(Y^a | L = \ell) \end{aligned}$$

- If the treatment effect varies across sub-population, we say there is **treatment effect heterogeneity**

$$\begin{aligned} E(Y^{a=1} | L = 55+) - E(Y^{a=0} | L = 55+) \\ \neq \\ E(Y^{a=1} | L = < 55) - E(Y^{a=0} | L = < 55) \end{aligned}$$

Conditional randomization

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 - ▶ If $Y^{a=1}$ has higher variability in some sub-population, assign more units to treated group

Conditional randomization

- ▶ Can be useful in designing experiments
 - ▶ If $Y^{a=1}$ has higher variability in some sub-population, assign more units to treated group
- ▶ Most useful as an idealized experiment to target with observational analysis
- ▶ Marginal exchangeability is very unlikely in observational data
- ▶ Conditional exchangeability may be more reasonable

Measures of association/causation¹

- ▶ For binary outcomes $Pr(Y^a = 1) = E(Y^a)$
- ▶ Average Causal Effect $E(Y^{a=1}) - E(Y^{a=0})$
- ▶ Also called average treatment effect and causal risk difference
- ▶ No average causal effect if $ACE = 0$

¹Ch 1.2 and 1.3 of Hernan and Robins

Measures of association/causation¹

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- ▶ Average Causal Effect $E(Y^{a=1}) - E(Y^{a=0})$
- ▶ Also called average treatment effect and causal risk difference
- ▶ No average causal effect if $ACE = 0$
- ▶ **Sharp null hypothesis:** $Y_i^{a=1} = Y_i^{a=0}$ for all i
- ▶ Sharp null hypothesis also means $ACE = 0$, but not the other way around!

¹Ch 1.2 and 1.3 of Hernan and Robins

Measures of association/causation²

- ▶ Causal Risk Ratio:

$$\frac{E(Y^{a=1})}{E(Y^{a=0})}$$

- ▶ Causal Odds Ratio:

$$\frac{Pr(Y^{a=1} = 1)/Pr(Y^{a=1} = 0)}{Pr(Y^{a=0} = 1)/Pr(Y^{a=0} = 0)}$$

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$$\frac{Pr(Y^{a=1} = 1)/Pr(Y^{a=1} = 0)}{Pr(Y^{a=0} = 1)/Pr(Y^{a=0} = 0)}$$

- ▶ No average causal effect if $CRR = COR = 1$

²Ch 1.2 and 1.3 of Hernan and Robins

Measures of association/causation³

- ▶ All measures will agree if $E(Y^{a=1}) = E(Y^{a=0})$
- ▶ If $E(Y^{a=1}) \neq E(Y^{a=0})$, the different measures may be easier/harder to interpret
- ▶ What is the ACE and CRR if
 - ▶ $E(Y^{a=1}) = .5; E(Y^{a=0}) = .25$
 - ▶ $E(Y^{a=1}) = .001; E(Y^{a=0}) = .0005$

³Ch 1.2 and 1.3 of Hernan and Robins

Standardization

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$$\begin{aligned} E(Y^a) &= Pr(L = 1)E(Y^a \mid L = 1) \\ &\quad + Pr(L = 0)E(Y^a \mid L = 0) \\ &= Pr(L = 1)E(Y \mid L = 1, A = a) \\ &\quad + Pr(L = 0)E(Y \mid L = 0, A = a) \end{aligned}$$

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- ▶ **Standardization** allows us to estimate the ACE by combining estimates from each sub-population
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- ▶ More generally, for each a

$$E(Y^a) = \sum_{\ell} Pr(L = \ell)E(Y | L = \ell, A = a)$$

Standardization

$$E(Y^a) = \sum_{\ell} E(Y | L = \ell, A = a) Pr(L = \ell)$$

Excercise

$$E(Y^a) = \sum_{\ell} E(Y | L = \ell, A = a)Pr(L = \ell)$$

		L	A	Y
1	Rheia	0	0	0
2	Kronos	0	0	1
3	Demeter	0	0	0
4	Hades	0	0	0
5	Hestia	0	1	0
6	Poseidon	0	1	0
7	Hera	0	1	0
8	Zeus	0	1	1

		L	A	Y
9	Artemis	1	0	1
10	Apollo	1	0	1
11	Leto	1	0	0
12	Ares	1	1	1
13	Athena	1	1	1
14	Hephaestus	1	1	1
15	Aphrodite	1	1	1
16	Polyphemus	1	1	1
17	Persephone	1	1	1
18	Hermes	1	1	0
19	Hebe	1	1	0
20	Dionysus	1	1	0

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$$E(Y^a) = Pr(L = 1)E(Y | L = 1, A = a)$$

$$+ Pr(L = 0)E(Y | L = 0, A = a)$$

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