

# Difference in differences: Extensions

INFO/STSCI/ILRST 3900: Causal Inference

7 Nov 2024

# Logistics

- ▶ Problem set 5 due tonight
  - ▶ Only one coding problem
  - ▶ Continuity/Smoothness assumption: **potential** outcomes are smooth/continuous **at the cutoff**
- ▶ Pset 5 peer reviews released Monday, due Friday
- ▶ Pset 6 released next Thurs, due following Thurs (no peer reviews)
- ▶ Final Project
  - ▶ Submit check-in by Sunday Nov 17th
  - ▶ Final paper due Dec 5
  - ▶ Video due Dec 18th (asynchronously)

# Learning goals for today

At the end of class, you will be able to:

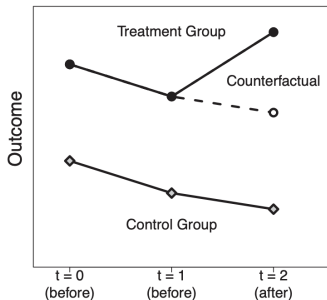
1. Use pre-treatment periods to
  - ▶ assess underlying assumptions
  - ▶ improve estimation accuracy
  - ▶ allow for a more flexible parallel trends assumption
2. recognize that the parallel trends assumption remains untestable
3. and compare the differences between parallel trends, *extended* parallel trends, and parallel *trends-in-trends*

Egami, N., & Yamauchi, S. (2023). [Using multiple pretreatment periods to improve difference-in-differences and staggered adoption designs](#). *Political Analysis*, 31(2), 195-212.

# PollEv: Parallel Trends Review

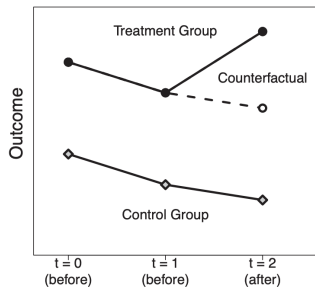
Parallel trends is...  
(select all that apply)

1. an assumption in the post-treatment period
2. an assumption about the treatment group
3. an assumption about a counterfactual
4. untestable



[PollEv.com/causal3900](https://PollEv.com/causal3900)

# Difference in difference

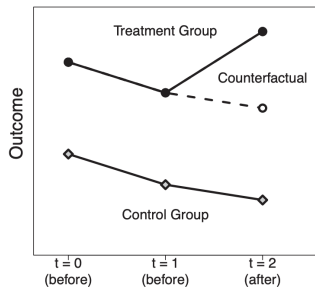


## Notation

$Y_{(group),(time)}^{treatment\ value}$

Example:  $Y_{Treated,1}^0$   
is outcome of treated group  
at time 1 under treatment 0

# Difference in difference



## Notation

$Y_{(group),(time)}^{\text{treatment value}}$

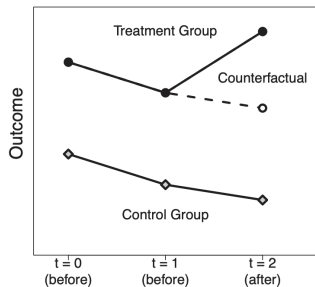
Example:  $Y_{Treated,1}^0$   
is outcome of treated group  
at time 1 under treatment 0

Parallel Trends Assumption  
(untestable)

---

$$E(Y_{Treated,2}^0 - Y_{Treated,1}^0) \\ = \\ E(Y_{Control,2}^0 - Y_{Control,1}^0)$$

# Difference in difference



## Notation

$Y_{(group),(time)}^{treatment\ value}$

Example:  $Y_{Treated,1}^0$   
is outcome of treated group  
at time 1 under treatment 0

Parallel Trends Assumption  
(untestable)

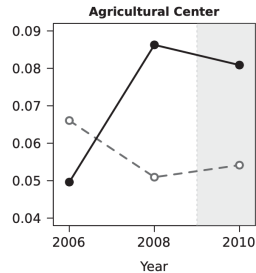
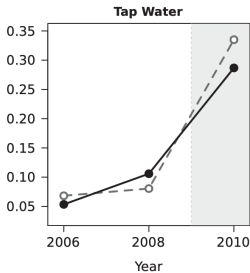
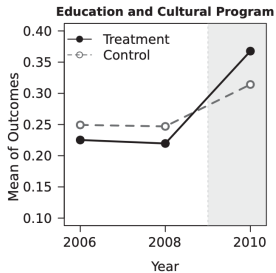
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$$E(Y_{Treated,2}^0 - Y_{Treated,1}^0) \\ = \\ E(Y_{Control,2}^0 - Y_{Control,1}^0)$$

Extended Parallel Trends  
(testable)

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$$E(Y_{Treated,1}^0 - Y_{Treated,0}^0) \\ = \\ E(Y_{Control,1}^0 - Y_{Control,0}^0)$$



In each case, do you believe parallel trends?



[PollEv.com/causal3900](https://PollEv.com/causal3900)



# Benefit 1: Assessing assumptions

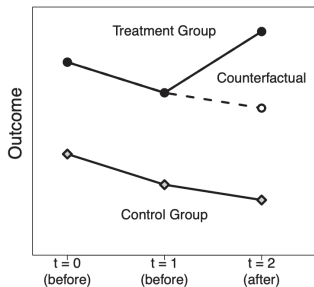
Pre-treatment periods enable us to  
**assess underlying assumptions**

Parallel trends is untestable, but being parallel  
in the pre-treatment period builds confidence

## Benefit 2: Improving efficiency

Pre-treatment periods also enable us to **improve estimation accuracy** when parallel trends holds

## Benefit 2: Improving efficiency



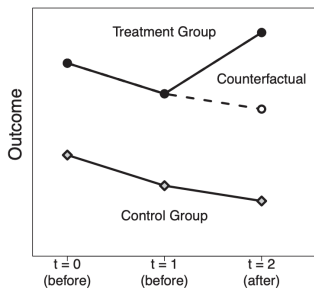
Estimator 1

Estimator 2

### Notation

$Y^{\text{treatment value}}$   
 $(\text{unit})(\text{time})$

## Benefit 2: Improving efficiency



Estimator 1

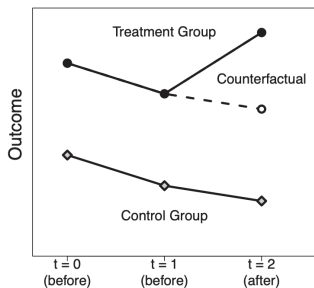
$$\underbrace{(\bar{Y}_{T2}^1 - \bar{Y}_{T1}^0)}_{\text{Treatment Group Time 2 - Time 1}} - \underbrace{(\bar{Y}_{C2}^0 - \bar{Y}_{C1}^0)}_{\text{Control Group Time 2 - Time 1}}$$

Estimator 2

Notation

$Y_{(\text{unit})(\text{time})}^{\text{treatment value}}$

## Benefit 2: Improving efficiency



Estimator 1

$$\underbrace{(\bar{Y}_{T2}^1 - \bar{Y}_{T1}^0)}_{\text{Treatment Group Time 2 - Time 1}} - \underbrace{(\bar{Y}_{C2}^0 - \bar{Y}_{C1}^0)}_{\text{Control Group Time 2 - Time 1}}$$

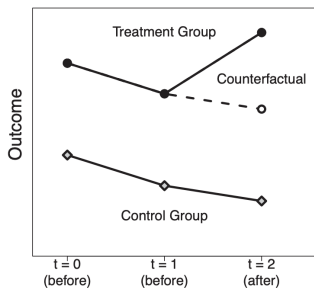
Estimator 2

$$\underbrace{(\bar{Y}_{T2}^1 - \bar{Y}_{T0}^0)}_{\text{Treatment Group Time 2 - Time 0}} - \underbrace{(\bar{Y}_{C2}^0 - \bar{Y}_{C0}^0)}_{\text{Control Group Time 2 - Time 0}}$$

### Notation

$\bar{Y}_{(\text{unit})}^{\text{treatment value}}(\text{time})$

## Benefit 2: Improving efficiency



Estimator 1

$$\underbrace{(\bar{Y}_{T2}^1 - \bar{Y}_{T1}^0)}_{\text{Treatment Group Time 2 - Time 1}} - \underbrace{(\bar{Y}_{C2}^0 - \bar{Y}_{C1}^0)}_{\text{Control Group Time 2 - Time 1}}$$

Estimator 2

$$\underbrace{(\bar{Y}_{T2}^1 - \bar{Y}_{T0}^0)}_{\text{Treatment Group Time 2 - Time 0}} - \underbrace{(\bar{Y}_{C2}^0 - \bar{Y}_{C0}^0)}_{\text{Control Group Time 2 - Time 0}}$$

Notation

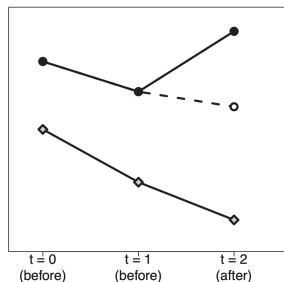
$\bar{Y}^{\text{treatment value}}$   
(unit)(time)

**Double DID Estimator:**  
**Average the two!**

## Benefit 3: A more flexible assumption

Pre-treatment periods make it possible to  
**allow for a more flexible parallel trends assumption**

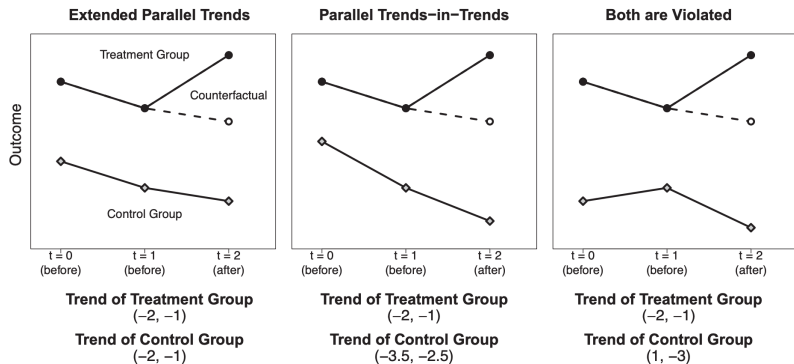
Parallel Trends-in-Trends



**Trend of Treatment Group**  
(-2, -1)

**Trend of Control Group**  
(-3.5, -2.5)

# Benefit 3: A more flexible assumption

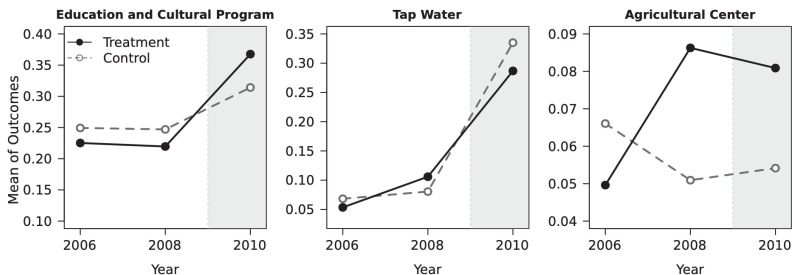


Egami, N., & Yamauchi, S. (2023). [Using multiple pretreatment periods to improve difference-in-differences and staggered adoption designs](#). *Political Analysis*, 31(2), 195-212.



# Benefits of multiple pre-treatment periods

1. assess underlying assumptions
2. improve estimation accuracy
3. allow for a more flexible parallel trends assumption



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