Regression Discontinuity: Extensions

INFO/STSCI/ILRST 3900: Causal Inference

26 Oct 2023

Learning goals for today

At the end of class, you will be able to:

1. Explain how fuzzy discontinuities can be used for causal identification
2. Understand how manipulation or other discontinuities may post potential threats to identification

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- Above the the cut-off $\mathrm{E}(Y^{a=1} \mid R = r) = \mathrm{E}(Y \mid R = r)$
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- Using observed data, estimate, $\mathrm{E}(Y \mid R = r)$ for r closer and closer to the cut-off
- Estimate local ATE $\mathrm{E}(Y_i^{a=1} \mid R_i = c) - \mathrm{E}(Y_i^{a=0} \mid X_i = c)$ by

$$\underbrace{lim_{x \to c^+} \mathrm{E}(Y \mid X = x)}_{\text{from above the cut-off}} - \underbrace{\lim_{x \to c^-} \mathrm{E}(Y \mid X = x)}_{\text{from below the cut-off}}$$

What can go wrong?

- **Other discontinuity**: Something other than treatment also jumps at the threshold
- **Fuzzy RDD**: Some units are treated on either side of threshold
- **Manipulation**: Units have control over over their running variable

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- National Merit example:
  - Suppose students above cut-off also receive tutoring on how to write better personal statements
  - Cannot distinguish between effect of Certificate of Merit and tutoring

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  - Ex: scores on writing sample)

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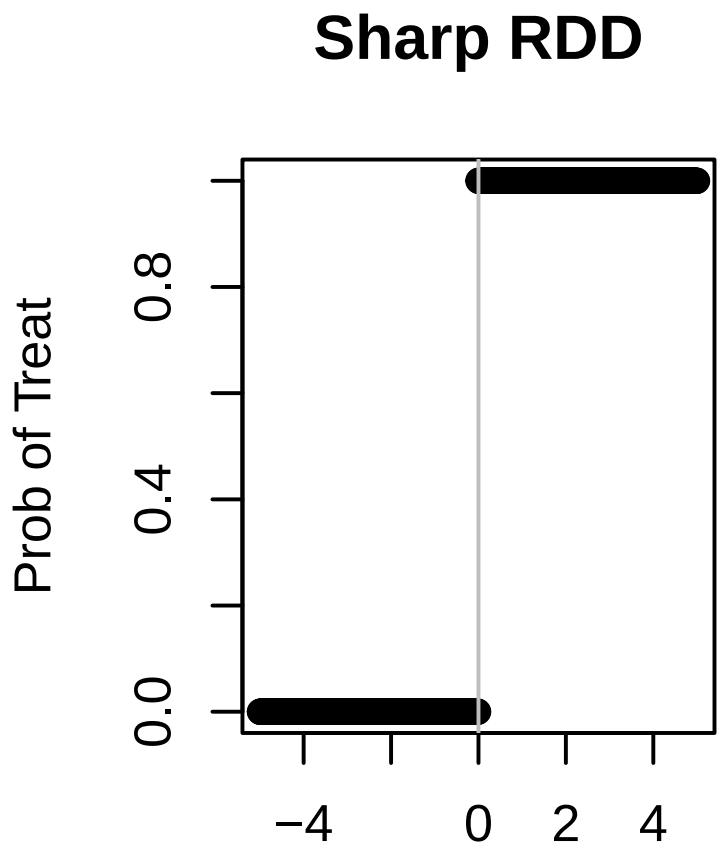

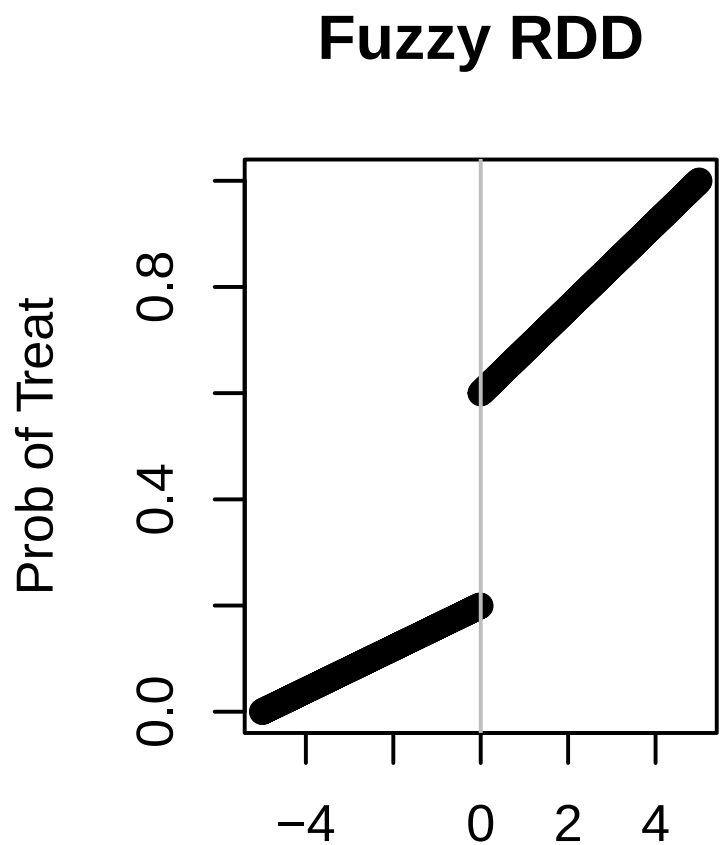

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- Effect of being above threshold on treatment

$$\lim_{r \to c^+} \mathrm{E}(A \mid R = r) - \lim_{r \to c^-} \mathrm{E}(A \mid R = r)$$

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$$\lim_{r \to c^+} \mathrm{E}(A \mid R = r) - \lim_{r \to c^-} \mathrm{E}(A \mid R = r)$$

- Dividing gives us the **local ATE for compliers**:

$$\frac{\lim_{r \to c^+} \mathrm{E}(Y \mid R = r) - \lim_{x \to c^-} \mathrm{E}(Y \mid R = r)}{\lim_{r \to c^+} \mathrm{E}(A \mid R = r) - \lim_{r \to c^-} \mathrm{E}(A \mid R = r)}$$

Manipulation

[2]Evaluating the effect of an antidiscrimination law using a regression-discontinuity design. Hahn et al. (1999)

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  - Firms with 14 should be essentially the same as firms with 15 employees
  - Firms have direct control over how many employees they hire
  - Those wanting to avoid EEOC law may decide to stay under 15 employees

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- Manipulation becomes a problem when units can choose precisely to be above/below threshold
- Units above the threshold are no longer essentially the same as units below the threshold

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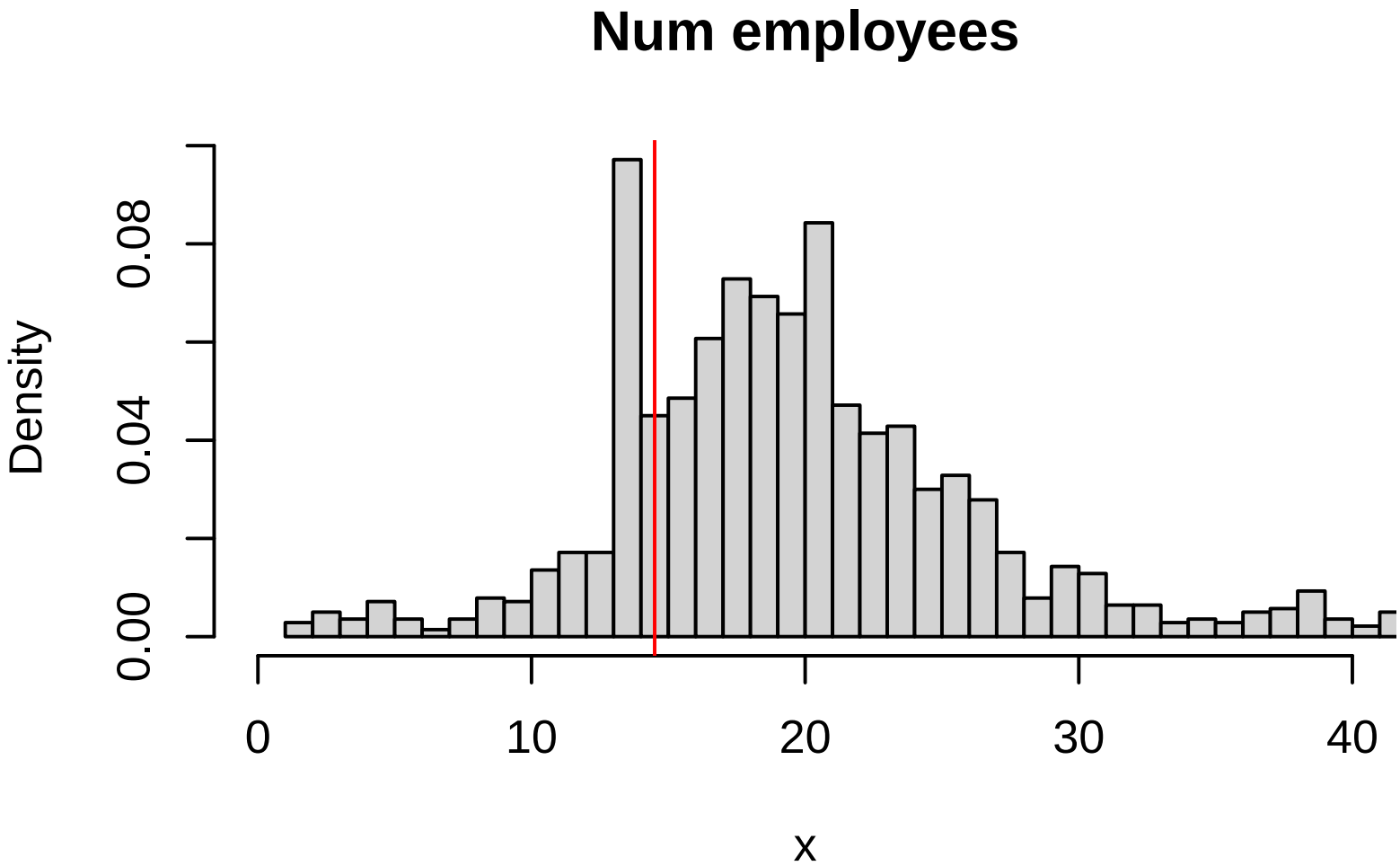
- If manipulation is occurring, we would expect to see "heaping" on one side of the cut-off



Code

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