

# Instrumental variables: Experiments with non-compliance

INFO/STSCI/ILRST 3900: Causal Inference

22 Oct 2024

# Logistics

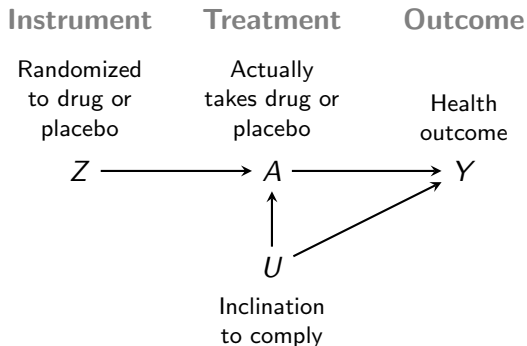
- ▶ No Task 2 peer review
- ▶ Peer reviews for Pset 3 due Oct 24 (Thurs)
- ▶ Pset 4 due Oct 25 (Friday)
- ▶ Walkthrough of code posted to canvas

# Learning goals for today

At the end of class, you will be able to:

1. Understand the logic of instrumental variables
2. Derive the average effect among compliers in experiments with noncompliance

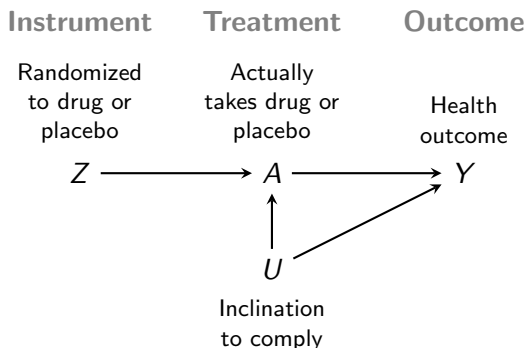
# Instrumental variables: Experiment with noncompliance



Two ideas

1. Intent to treat effect
2. Average effect among compliers

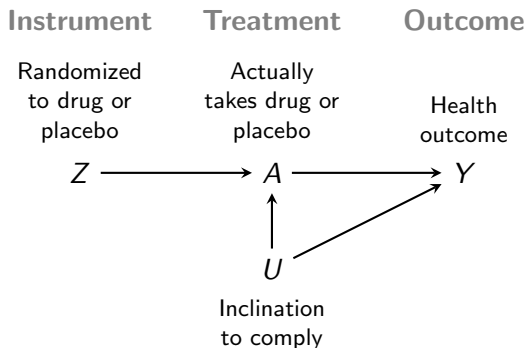
# Instrumental variables: 1) Intent to treat effect



Ignore  $A$ . What is the effect of  $Z$  on  $Y$ ?

$$\begin{array}{ccc}
 E(Y^{z=1} - Y^{z=0}) & = & E(Y \mid Z = 1) - E(Y \mid Z = 0) \\
 \uparrow & \uparrow & \\
 \text{Outcome} & \text{By} & \text{Mean difference in} \\
 \text{under} & \text{Positivity} & \text{observable outcomes} \\
 Z = 1 & \text{Consistency} & \\
 & \text{Exchangeability} & \\
 & \text{for } Z & 
 \end{array}$$

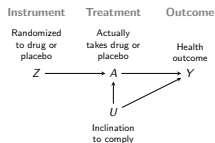
## Instrumental variables: 2) Average effect among compliers



**Key insight:** The effect of  $Z$  on  $Y$  operates entirely through  $A$

1. Study the effect of  $Z \rightarrow Y$  (we just did)
2. Study the effect of  $Z \rightarrow A$
3. Learn about  $A \rightarrow Y$  since  $Z \rightarrow Y$  is  $Z \rightarrow A \rightarrow Y$

## Instrumental variables: 2) Average effect among compliers



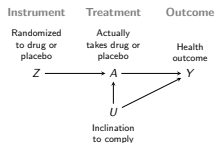
The effect  $Z \rightarrow A$  has four **principal strata**:  
latent sets of people who respond to  $Z$  a particular way

Compliers	$A^{Z=0} = 0$	$A^{Z=1} = 1$	(follow assignment)
Always takers	$A^{Z=0} = 1$	$A^{Z=1} = 1$	(always take treatment)
Never takers	$A^{Z=0} = 0$	$A^{Z=1} = 0$	(never take treatment)
Defiers	$A^{Z=0} = 1$	$A^{Z=1} = 0$	(defy assignment)

Fundamental problem of causal inference applies to  $Z \rightarrow A$

**Discuss:** In which strata is the effect  $Z \rightarrow Y$  zero?

## Instrumental variables: 2) Average effect among compliers



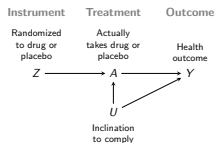
Among **always takers** and **never takers**,  
 $Z$  does not affect  $A$

$Z$  only affects  $Y$  through  $A$

In these strata,  $Z$  does not affect  $Y$



## Instrumental variables: 2) Average effect among compliers



Among **compliers**,

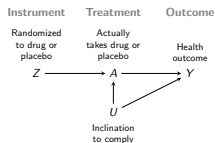
$Z = 1$  implies  $A = 1$  and

$Z = 0$  implies  $A = 0$

In these strata,  $Z = A$

$Z \rightarrow Y$  and  $A \rightarrow Y$  are the same

## Instrumental variables: 2) Average effect among compliers



Among **defiers**,

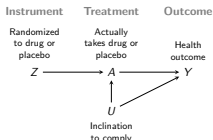
$Z = 1$  implies  $A = 0$  and

$Z = 0$  implies  $A = 1$

In these strata,  $Z = 1 - A$

$Z \rightarrow Y$  and  $A \rightarrow Y$  are the same magnitude  
but have opposite signs

## Instrumental variables: 2) Average effect among compliers



Four principal strata

Compliers  $(Z \rightarrow A) = +1$   $(Z \rightarrow Y) = (A \rightarrow Y)$

Always takers  $(Z \rightarrow A) = 0$   $(Z \rightarrow Y) = 0$

Never takers  $(Z \rightarrow A) = 0$   $(Z \rightarrow Y) = 0$

Defiers  $(Z \rightarrow A) = -1$   $(Z \rightarrow Y) = -(A \rightarrow Y)$  Defiers

Assume **no defiers** in the population

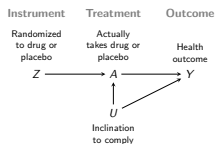
**Discuss a hypothetical.**

Population is 50% compliers, 25% always takers, 25% never takers

Average effect of  $Z \rightarrow Y$  among compliers is 0.6

What is the average effect of  $Z \rightarrow Y$  in the population?

## Instrumental variables: 2) Average effect among compliers



### Four principal strata

Compliers	$(Z \rightarrow A) = +1$	$(Z \rightarrow Y) = (A \rightarrow Y)$
Always takers	$(Z \rightarrow A) = 0$	$(Z \rightarrow Y) = 0$
Never takers	$(Z \rightarrow A) = 0$	$(Z \rightarrow Y) = 0$
Defiers	$(Z \rightarrow A) = -1$	$(Z \rightarrow Y) = -(A \rightarrow Y)$

Assume **no defiers** in the population

**Discuss a hypothetical.**

Population is 50% compliers, 25% always takers, 25% never takers

Average effect of  $Z \rightarrow Y$  among compliers is 0.6

What is the average effect of  $Z \rightarrow Y$  in the population?

Could you calculate the proportion of compliers in the population?

## Instrumental variables: 2) Average effect among compliers

Four principal strata

Compliers	$(Z \rightarrow A) = +1$	$(Z \rightarrow Y) = (A \rightarrow Y)$	
Always takers	$(Z \rightarrow A) = 0$	$(Z \rightarrow Y) = 0$	(1)
Never takers	$(Z \rightarrow A) = 0$	$(Z \rightarrow Y) = 0$	

**Discuss a hypothetical.**

Population is 50% compliers, 25% always takers, 25% never takers

Average effect of  $Z \rightarrow Y$  among compliers is 0.6

What is the average effect of  $Z \rightarrow Y$  in the population?

Could you calculate the proportion of compliers in the population?



## Instrumental variables: Proportion of Compliers

$$\begin{aligned} E(A | Z = 1) - E(A | Z = 0) &= E(A^{Z=1} - A^{Z=0}) \\ &= \sum_s E(A^{Z=1} - A^{Z=0} | S = s) \underbrace{P(S = s)}_{\substack{\text{Denote} \\ \pi_s}} \\ &= E(A^{Z=1} - A^{Z=0} | S = \text{Complier})\pi_{\text{Complier}} \\ &\quad + E(A^{Z=1} - A^{Z=0} | S = \text{Always-Taker})\pi_{\text{Always-Taker}} \quad (= 0) \\ &\quad + E(A^{Z=1} - A^{Z=0} | S = \text{Never-Taker})\pi_{\text{Never-Taker}} \quad (= 0) \\ &\quad + E(A^{Z=1} - A^{Z=0} | S = \text{Defier})\pi_{\text{Defier}} \quad (= 0) \\ &= \pi_{\text{Complier}} \end{aligned} \tag{2}$$

Assuming no defiers,

$$\pi_{\text{Complier}} = E(A | Z = 1) - E(A | Z = 0)$$

## Instrumental variables: 2) Average effect among compliers

Deriving the general case:

$$\begin{aligned} E(Y | Z = 1) - E(Y | Z = 0) &= E(Y^{Z=1} - Y^{Z=0}) \\ &= \sum_s E(Y^{Z=1} - Y^{Z=0} | S = s) \underbrace{P(S = s)}_{\substack{\text{Denote} \\ \pi_s}} \\ &= E(Y^{Z=1} - Y^{Z=0} | S = \text{Complier})\pi_{\text{Complier}} && (3) \\ &\quad + E(Y^{Z=1} - Y^{Z=0} | S = \text{Always-Taker})\pi_{\text{Always-Taker}} && (= 0) \\ &\quad + E(Y^{Z=1} - Y^{Z=0} | S = \text{Never-Taker})\pi_{\text{Never-Taker}} && (= 0) \\ &\quad + E(Y^{Z=1} - Y^{Z=0} | S = \text{Defier})\pi_{\text{Defier}} && (= 0) \end{aligned}$$

## Instrumental variables: 2) Average effect among compliers

Average effect of instrument = Average among compliers  $\times$  Proportion who comply

$$E(Y^{Z=1} - Y^{Z=0}) = E(Y^{Z=1} - Y^{Z=0} \mid S = \text{Complier})\pi_{\text{Complier}}$$

Among compliers,  $(Z \rightarrow Y) = (A \rightarrow Y)$ .

$$E(Y^{Z=1} - Y^{Z=0}) = E(Y^{A=1} - Y^{A=0} \mid S = \text{Complier})\pi_{\text{Complier}}$$

Rearrange to get the complier average treatment effect

$$\begin{aligned} E(Y^{A=1} - Y^{A=0} \mid S = \text{Complier}) &= \frac{E(Y^{Z=1} - Y^{Z=0})}{\pi_{\text{Complier}}} \\ &= \frac{E(Y \mid Z = 1) - E(Y \mid Z = 0)}{E(A \mid Z = 1) - E(A \mid Z = 0)} \end{aligned} \quad (4)$$



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