Matching Intro

INFO/STSCI/ILRST 3900: Causal Inference

10 Oct 2024

Learning goals for today

At the end of class, you will be able to:

1. Explain how matching can be used to estimate causal effects
2. Explain bias variance trade-off in various matching procedures

Causal effect

What is the causal effect on income of a job training program?

- **Average Treatment Effect** (on everyone)

$$E(Y^{a=1}) - E(Y^{a=0})$$

- **Average Treatment Effect on the Treated** (ATT)

$$E(Y^{a=1} \mid A = 1) - E(Y^{a=0} \mid A = 1)$$

Matching: The big idea

Goal: $\mathrm{E}(Y^{a=1} \mid A=1) - \mathrm{E}(Y^{a=0} \mid A=1)$ **ATT**

$$\mathrm{E}(Y^{a=1} \mid A=1) \approx \frac{1}{n_t} \sum_{i:A_i=1} Y_i^{a=1} = \frac{1}{n_t} \sum_{i:A_i=1} Y_i$$

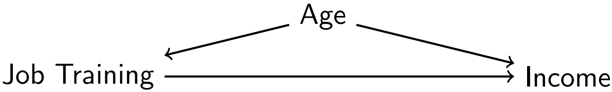
$$\mathrm{E}(Y^{a=0} \mid A=1) \approx \frac{1}{n_t} \sum_{i:A_i=1} Y_i^{a=0} \not\approx \frac{1}{n_c} \sum_{i:A_i=0} Y_i$$

Problem: Control may be different than the treatment

Potential Solution: Create a sample of **untreated** individuals, $\mathcal{M}$, which are similar to the treated group

$$\frac{1}{n_m} \sum_{i \in \mathcal{M}} Y_i = \frac{1}{n_m} \sum_{i \in \mathcal{M}} Y_i^{a=0} \approx \frac{1}{n_t} \sum_{i:A_i=1} Y_i^{a=0}$$

Example



- Conditional exchangeability holds when conditioning on Age!

$$\mathrm{E}(Y^{a=0} \mid A = 1, \mathrm{Age} = \ell) = \mathrm{E}(Y^{a=0} \mid A = 0, \mathrm{Age} = \ell)$$

- Estimate

$$\mathrm{E}(Y^{a=0} \mid A = 1) = \underbrace{\sum_{\ell} Pr(\mathrm{Age} = \ell \mid A = 1)\mathrm{E}(Y^{a=0} \mid A = 1, \mathrm{Age} = \ell)}_{\text{Weighted average of averages}}$$

$$\begin{aligned}\mathrm{E}(Y^{a=0} \mid \mathcal{M}) &= \textstyle\sum_{\ell} Pr(\mathrm{Age} = \ell \mid \mathcal{M})\mathrm{E}(Y^{a=0} \mid A = 0, \mathrm{Age} = \ell, \mathcal{M}) \\ &= \textstyle\sum_{\ell} Pr(\mathrm{Age} = \ell \mid \mathcal{M})\mathrm{E}(Y^{a=0} \mid A = 0, \mathrm{Age} = \ell)\end{aligned}$$

- If we can make $Pr(\mathrm{Age} = \ell \mid \mathcal{M}) \approx Pr(\mathrm{Age} = \ell \mid A = 1)$, the two quantities should be the same

Matching: The big idea

Goal: Sample Average Treatment Effect on the Treated

$$E(Y^{a=1} \mid A = 1) - E(Y^{a=0} \mid A = 1)$$

Potential Solution: Create a group of untreated individuals, $\mathcal{M}$, which have a **similar distribution of** L to the treated group

$$\frac{1}{n_m} \sum_{i \in \mathcal{M}} Y_i \approx \frac{1}{n_t} \sum_{i:A_i=1} Y_i^{a=0} \approx E(Y^{a=0} \mid A = 1)$$

Detail: How?

Example

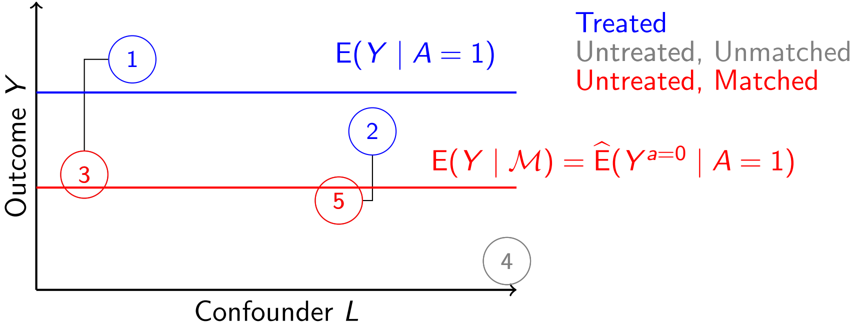
Job training

Ind	Age	Y^{Train}	Y^{NoTrain}
1	20	19	?
2	25	63	?
3	38	65	?
4	38	43	?

No job training

Ind	Age	Y^{NoTrain}
1	19	82
2	18	39
3	20	49
4	20	56
5	24	33
6	26	82
7	26	35
8	38	35
9	28	83
10	30	79
11	24	63
12	32	52
13	34	58
14	34	70
15	35	47
16	37	42
17	37	83
18	38	33
19	39	37
20	39	60

Matching: The big idea



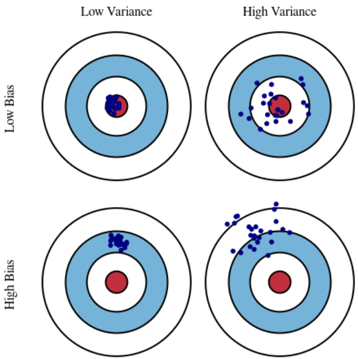
You have a some treated units.

Why matching is great

1. Completely transparent that Y_i^1 is observed
2. Easy to explain
   - We had some treated units
   - We found a set of control units which are comparable
   - We compared the means
3. Can assess quality of matches before we look at the outcome
4. Model-free*
   - * but you have to define what makes a match "good"

Bias vs variance

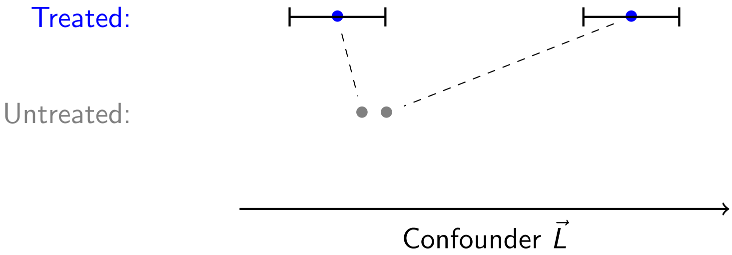
The idea of matching is straightforward, but the details matter!



Matching in univariate settings: Algorithms

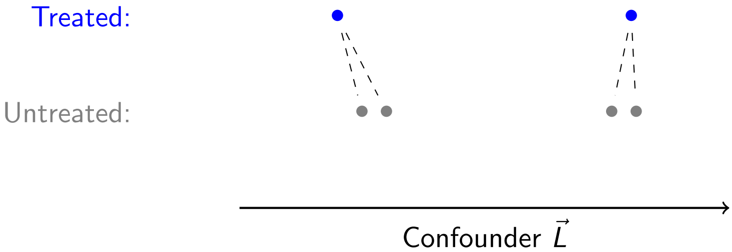
- Caliper or no caliper
- 1:1 vs k:1
- With replacement vs without replacement
- Greedy vs optimal

Caliper or no caliper matching



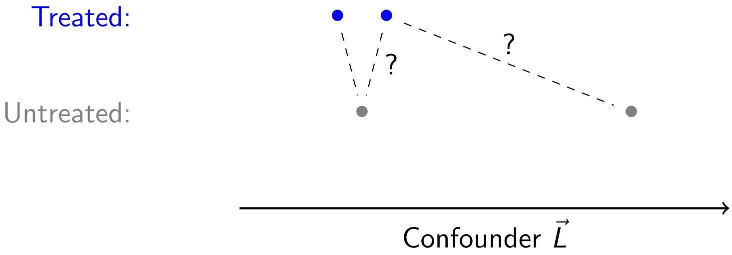
- Caliper: A radius around a treated unit such that we would rather drop the unit than make a match beyond that radius
- Feasible Sample Average Treatment Effect on the Treated (FSATT): Average among treated units for whom an acceptable match exists

1:1 vs k:1 matching



- Benefit of 2:1 matching
  - Lower variance. Averaging over more cases.
- Benefit of 1:1 matching
  - Lower bias. Only the best matches.
- Greater $k \rightarrow$ lower variance, higher bias

With replacement vs without replacement matching

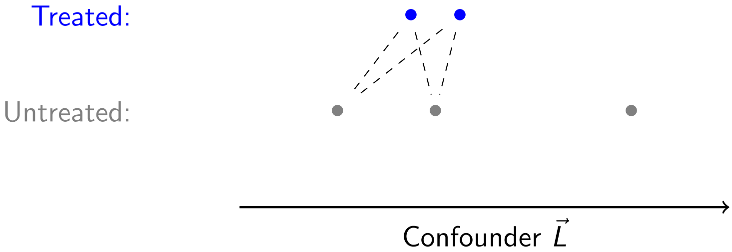


- Benefit of matching without replacement
  - Lower variance. Averaging over more cases.
- Benefit of matching with replacement
  - Lower bias. Better matches.

Greedy vs optimal matching[1]

Greedy Matching:
Match sequentially

Optimal Matching:
Consider the whole set of matches



- Optimal is better. Just computationally harder.

[1] Gu, X. S., & Rosenbaum, P. R. (1993). Comparison of multivariate matching methods: Structures, distances, and algorithms. Journal of Computational and Graphical Statistics, 2(4), 405-420.

Matching in univariate settings: Algorithms

- Caliper or no caliper
- 1:1 vs k:1
- With replacement vs without replacement
- Greedy vs optimal

Many reasonable choices, good choices depend on the data you have

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