

Causal Discovery

STSCI/INFO/ILRST 3900: Causal Inference

Oct 6 2026

Logistics

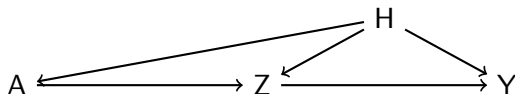
- ▶ Quiz 2 on Thursday in class (accommodations through ATP)
- ▶ Bring pencil/pen
- ▶ Review practice questions in discussion section Wednesday

FAQs

Causal paths are paths that follow the direction of the arrows

- ▶ Does not depend on conditioning set
- ▶ Different characteristic than open/closed (i.e., can be open causal, open non-causal, blocked causal, blocked non-causal)

Practice



► $A \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$

causal path

► $A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$

non-causal

► $A \rightarrow \underbrace{Z}_{\text{Col}} \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$

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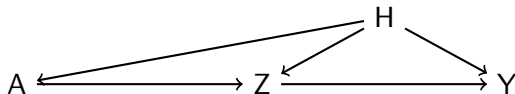
non-causal

FAQs

Conditional Exchangeability depends on checking the entire graph, not just individual paths

- ▶ Conditional exchangeability holds if: all open paths are causal paths
- ▶ Conditional exchangeability does not hold if: any open paths are non-causal

Practice



When conditioning on H :

- ▶ $A \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$ causal and open
- ▶ $A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$ non-causal and blocked
- ▶ $A \rightarrow \underbrace{Z}_{\text{Col}} \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$ non-causal and blocked
- ▶ $A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$ non-causal and blocked

Conditional exchangeability holds when conditioning on H

FAQs

A specific node can be a collider on one path and a non-collider on another



$$\blacktriangleright A \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$$

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Causal Discovery

- ▶ So far, we have assumed the DAG is known from expert knowledge
- ▶ DAG tells us about conditional independence we would observe in data

DAG $\Rightarrow$ Conditional independence in data

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- ▶ DAG tells us about conditional independence we would observe in data

DAG $\Rightarrow$ Conditional independence in data

- ▶ Conditional independence is a observational quantity (i.e., not causal)
- ▶ Can be tested in observed data
- ▶ Can we go in the opposite direction?

Conditional independence in data $\stackrel{?}{\Rightarrow}$ DAG

Causal Discovery: biological systems

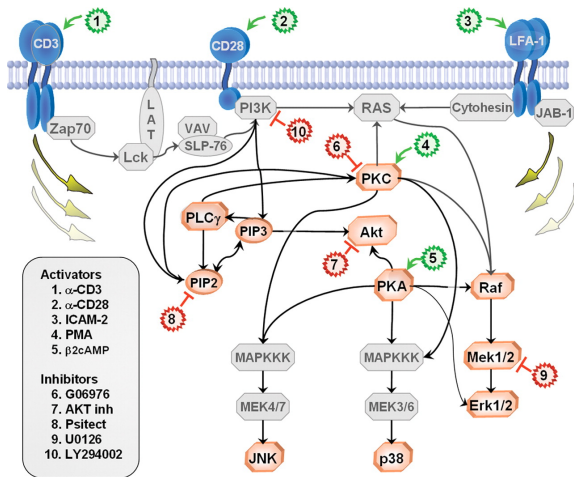


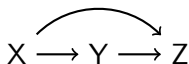
Figure: Regulatory network from Sachs et al (2005, Science)

Causal Discovery

Can we tell which nodes are/aren't connected by an edge?

Causal Discovery

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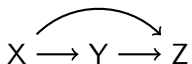
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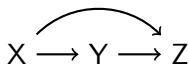
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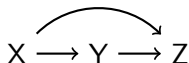
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If there is an edge between two nodes, they cannot be made conditionally independent!

Rule 1

- ▶ Start with (undirected) edges between every pair of nodes
- ▶ If you can find a set L such that $X \perp\!\!\!\perp Y \mid L$, take away the edge between X and Y

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A **skeleton** is the DAG where we have made all edges undirected

DAG : $X \rightarrow Y \rightarrow Z$

Skeleton : $X - Y - Z$

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Colliders can sometimes tell us the direction of an edge

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- ▶ Suppose we have $X - Y - Z$ and no edge between X and Z
- ▶ Suppose $X \perp\!\!\!\perp Z \mid L$ for some set L that does not contain Y

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- ▶ So it must be $X \rightarrow Y \leftarrow Z$
- ▶ **Unshielded collider:** $X \rightarrow Y \leftarrow Z$ and X and Z do not have an edge

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Some graphs have the exact same set of conditional independence statements and cannot be distinguished from data alone!

Rule 3

Graphs imply the same conditional independence statements if

- ▶ Same skeleton: edges in the same location, but possibly different direction (from Rule 1)
- ▶ Same unshielded colliders: $X \rightarrow Y \leftarrow Z$ and X and Z do not share an edge (from Rule 2)