Inverse Probability Weighting

INFO/STSCI/ILRST 3900: Causal Inference

12 Sep 2024

Logistics

- Peer reviews assigned today @ 12pm, due Tue (9/17) @ 5pm
  - Post on Ed Discussion explaining everything
  - **This link** will give you a sense of what to expect
- Pset 2 released by Tue (9/17) and due Tue (9/24) @ 5pm
- Post questions on Ed Discussion or come to office hours!
  - Filippo: **Mon** 11am-12pm (Comstock 1187)
  - Sam: **Tues** 4-5pm (Comstock 1187)
  - Shira: **Wed** 5:30-6:30pm (Comstock 1187)
  - Mayleen: **Thur** 11am-12pm (Rhodes 657, Room 1)
- After class, read 3.1 and 3.2 of Hernán & Robins

Learning goals for today

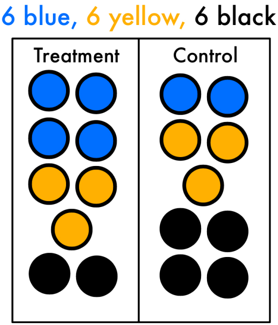
At the end of class, you will be able to:

1. Estimate the average treatment effect using inverse probability weighting
2. Explain the challenge of satisfying conditional exchangeability in observational data

Check Your Understanding: Exchangeability

Discuss in groups, then submit your response individually to PollEverywhere. Your response won't be graded.

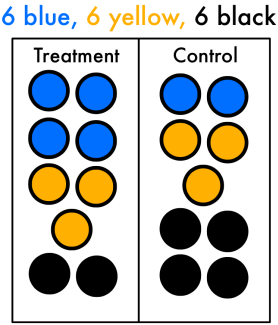
Consider a study analyzing the effect of a college degree on income level. The colors below represent **parental** education (blue: college degree, yellow: some college, black: high school). Based on the data below, do you think exchangeability holds? Why or why not?



https://pollev.com/causal3900

PollEverywhere Choices

Consider a study analyzing the effect of a college degree on income level. The colors below represent **parental** education (blue: college degree, yellow: some college, black: high school). Based on the data below, do you think exchangeability holds? Why or why not?



(A) Marginal exchangeability holds

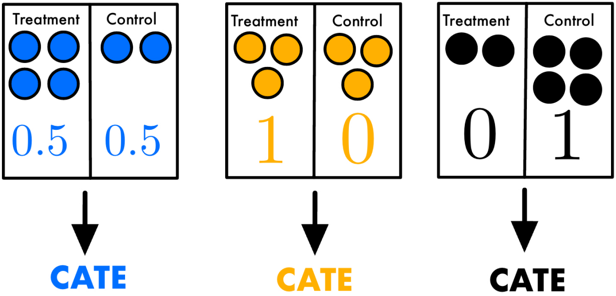
(B) Exchangeability holds conditional on parental education (the colors)

(C) It depends–more info is needed to determine if exchangeability holds

(D) Neither marginal nor conditional exchangeability hold

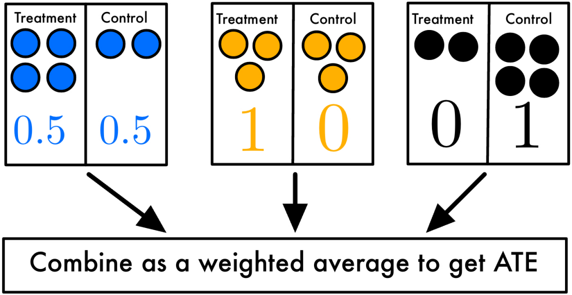
Review: Stratification

- **Conditional exchangeability:** $Y^a \perp\!\!\!\perp A \mid L$ for all a
- **Stratification**: estimate the conditional average treatment effect (CATE) by looking within each stratum



Review: Standardization

- **Conditional exchangeability:** $Y^a \perp\!\!\!\perp A \mid L$ for all a
- **Standardization**: estimate the population average treatment effect (ATE) by taking a weighted average across strata
  - $E(Y^{a=1}) = \sum_\ell E(Y \mid L = \ell, A = 1) Pr(L = \ell)$
  - $E(Y^{a=0}) = \sum_\ell E(Y \mid L = \ell, A = 0) Pr(L = \ell)$



$$E(Y^{a=\text{treatment}}) = 0.5 \cdot \frac{6}{18} + 1 \cdot \frac{6}{18} + 0 \cdot \frac{6}{18}$$
$$E(Y^{a=\text{control}}) = 0.5 \cdot \frac{6}{18} + 0 \cdot \frac{6}{18} + 1 \cdot \frac{6}{18}$$

Inverse probability weighting

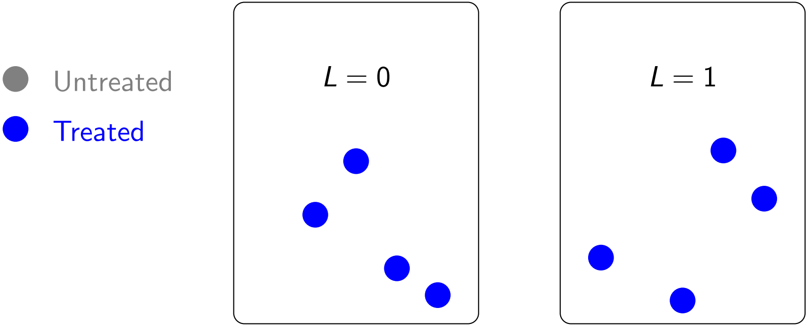
- Standardization: constructs an estimate of $\mathrm{E}(Y^a)$ through a weighted average
- Inverse probability weighted (IPW) estimator is equivalent to standardization
- Estimator for the population expected potential outcome

$$\mathrm{E}(Y^a) = \frac{1}{N} \sum_{i:A_i=a} \frac{Y_i}{\pi_i^a}$$

- $\pi_i^a = \mathrm{P}(A_i = a \mid L = \ell_i)$ is the probability of the observed treatment conditioning on confounders
- N is the total number of observations

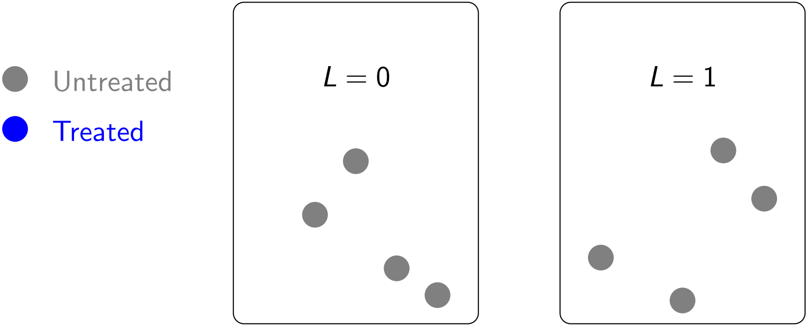
$$\widehat{ATE}_{\mathrm{IPW}} = \frac{1}{N} \sum_{i:A_i=1} \frac{Y_i}{\pi_i^1} \; - \; \frac{1}{N} \sum_{i:A_i=0} \frac{Y_i}{\pi_i^0}$$

Inverse probability weighting: Conditional randomizaton



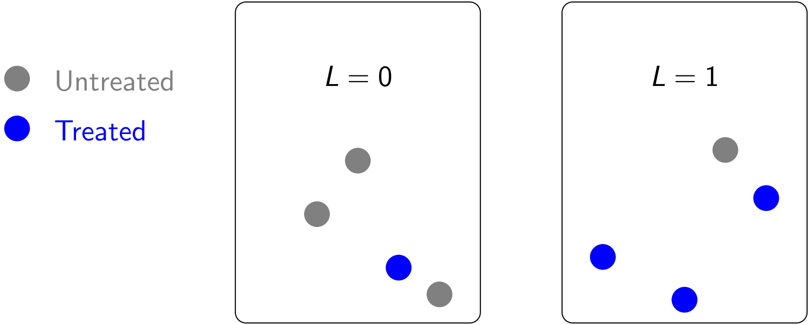
Hypothetical world where everyone is treated

Inverse probability weighting: Conditional randomizaton



Hypothetical world where no-one is treated

Inverse probability weighting: Conditional randomizaton



$$\pi_i^a = \text{P}(A_i = a \mid L_i) : \quad \begin{cases} \frac{1}{4} & \text{if } a = 1 \\ \frac{3}{4} & \text{if } a = 0 \end{cases} \qquad \begin{cases} \frac{3}{4} & \text{if } a = 1 \\ \frac{1}{4} & \text{if } a = 0 \end{cases}$$

$$\text{Each counts for:} \quad \begin{cases} \frac{4}{1} & \text{if } a = 1 \\ \frac{4}{3} & \text{if } a = 0 \end{cases} \qquad \begin{cases} \frac{4}{3} & \text{if } a = 1 \\ \frac{4}{1} & \text{if } a = 0 \end{cases}$$

Conditional exchangeability in observational data

- Conditional exchangeability let's us estimate causal effects
- Stratification: conditional average treatment effects
- Standardization or inverse probability weighting: population average treatment effect
- By design, conditional exchangeability holds in conditionally randomized experiments
- Conditional exchangeability more reasonable in observational data than marginal exchangeability

What could go wrong?

Data gathered by surveying individuals in Fall 2021

- Whether they were vaccinated for COVID
  $A_i = 1$ if vaccinated, $A_i = 0$ if not vaccinated
- Whether they tested positive for Covid in 2021
  $Y_i = 1$ if positive test, $Y_i = 0$ if no positive test
- Suppose vaccinated group had lower rates of positive COVID tests. How might a vaccine *skeptic* explain that?
- Suppose vaccinated group had higher rates of positive COVID tests. How might a vaccine *advocate* explain that?
- What extra information could you gather about each individual to make conditional exchangeability plausible?

$$Y^{a=1}, Y^{a=0} \perp\!\!\!\perp A \mid L$$

Conditional exchangeability in observational data

- Even if gathering data was possible for every covariate we want, when do we stop?
- Never 100% sure that conditional exchangeability holds
- Is it reasonable?

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