Stratification and Standardization

INFO/STSCI/ILRST 3900: Causal Inference

10 Sep 2023

Logistics

- Problem Set 1 due today by 5pm
- Peer reviews assigned via Canvas Thursday (9/12) @ 12pm and due Tuesday (9/17) @ 5pm
- Post questions on Ed Discussion or come to office hours!
  - Filippo: **Mon** 11am-12pm (Comstock 1187)
  - Sam: **Tue** 4-5pm (Comstock 1187)
  - Shira: **Wed** ~~3-4pm~~ **5:30-6:30pm** (Comstock 1187)
  - Mayleen: **Thu** ~~10:15-11:15am~~ **11am-12pm** (Rhodes 657)
- After class, read 2.4 of Hernán & Robins

Learning goals for today

At the end of class, you will be able to:

1. Explain marginal versus conditional exchangeability
2. Estimate the conditional average treatment effect using stratification
3. Estimate the average treatment effect using standardization

Check Your Understanding: Exchangeability

Discuss in groups, then submit your response individually to PollEverywhere. Your response won't be graded.

Registered voters are randomly assigned to two groups. The treatment group receives a phone call reminder to vote. The control group does not. After the election, we look at voter turnout in both groups. **What does exchangeability mean in this study?**

https://pollev.com/causal3900

PollEverywhere Answer

Registered voters are randomly assigned to two groups. The treatment group receives a phone call reminder to vote. The control group does not. After the election, we look at voter turnout in both groups. **What does exchangeability mean in this study?**

1. Assigned treatment (call or no call) has no causal effect on the observed outcome (vote or no vote).
2. Assigned treatment (call or no call) has no causal effect on the potential outcome (whether or not they would have voted).
3. Assigned treatment (call or no call) is independent of the observed outcome (vote or no vote).
4. **Assigned treatment (call or no call) is independent of the potential outcome (whether or not they would have voted).**

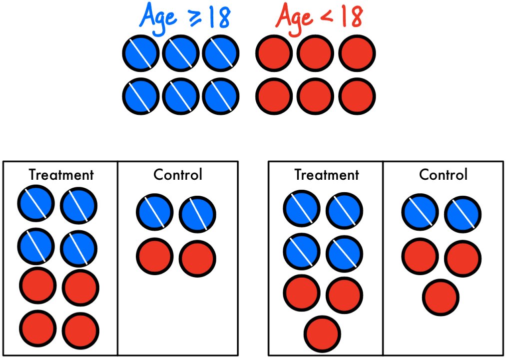
The only correct answer is number 4.

Review: conditional randomization

- Causal effect of job training on employment rate
- Setup 1: flip same coin for everyone (left)
  - Job training with probability 2/3
- Setup 2: flip different coins depending on age (right)
  - Age ≥ 18: Job training with probability 2/3
  - Age < 18: Job training with probability 1/2

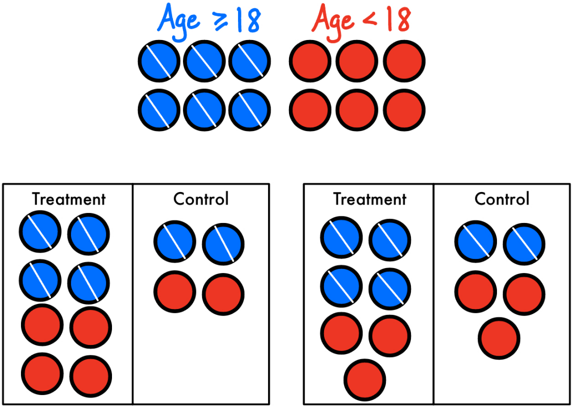
Observe: distribution of age groups is the same in treatment versus control in setup 1, but not setup 2

Key assumption: $Y^a \not\perp\!\!\!\perp$ AGE (L)



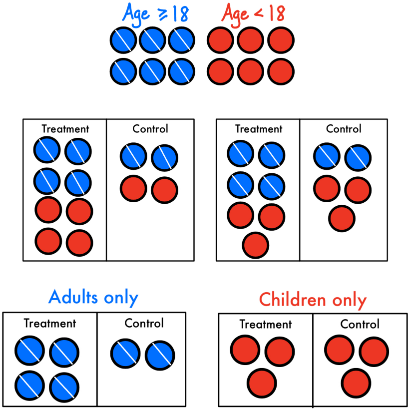
Review: conditional randomization

- **Marginal exchangeability:** $Y^a \perp\!\!\!\perp A$ for all a
- Suppose we find there is a treatment effect using the second setup. What is suspicious about that result?
- In practice, one way to reason about exchangeability is balance across covariates



Review: conditional randomization

- **Conditional exchangeability:** $Y^a \perp\!\!\!\perp A \mid L$ for all a
- When we look within sub-populations, or *strata*, marginal exchangeability holds



Check-in: Marginal vs Conditional Exchangeability

- **Marginal exchangeability:** $Y^a \perp\!\!\!\perp A$ for all a
- **Conditional exchangeability:** $Y^a \perp\!\!\!\perp A \mid L$ for all a

  The potential outcomes are independent of treatment **conditional on** L (e.g. L could be an indicator of age group)

- How do you feel about marginal versus conditional exchangeability (thumbs up or thumbs down)?
- What questions do you have?

Stratification

- Exchangeability holds *within* each sub-population
- **Stratification**: We can directly estimate causal effect within each sub-population (or stratum)
- The Conditional Average Treatment Effect (CATE)

$$CATE = E(Y^{a=1} \mid L = \ell) - E(Y^{a=0} \mid L = \ell)$$

- Conditional exchangeability (+ consistency) lets us estimate expected potential outcomes from observable averages

$$\begin{aligned} \mathrm{E}(Y \mid A = a, L = \ell) &\overset{\text{consis}}{=} \mathrm{E}(Y^a \mid A = a, L = \ell) \\ &\overset{\text{exchange}}{=} \mathrm{E}(Y^a \mid L = \ell) \end{aligned}$$

- If the treatment effect varies across sub-population, we say there is **treatment effect heterogeneity**

Stratification

- Estimate the Conditional Average Treatment Effect (CATE)
- $CATE = E(Y^{a=1} \mid L = \ell) - E(Y^{a=0} \mid L = \ell)$
- Example: medication

- Sometimes we do need the Average Treatment Effect (ATE)
- $ATE = E(Y^{a=1}) - E(Y^{a=0})$
- Example: new feature on YouTube (two thumbs up)
- How can we go from CATE to ATE?

- Law of Total Expectation:

$$E[Y^a] = \sum_{\ell} E[Y^a \mid L = \ell] \cdot Pr(L = \ell)$$

Standardization

- **Standardization** allows us to estimate the ATE by combining estimates from each sub-population
- For $L = 0, 1$

$$\mathrm{E}(Y^a) = \mathrm{E}(Y^a \mid L = 1) \cdot Pr(L = 1) \; + \; \mathrm{E}(Y^a \mid L = 0) \cdot Pr(L = 0)$$

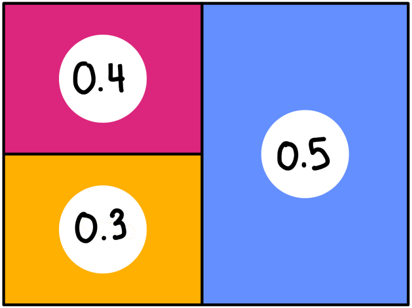
$$\begin{aligned} &= \mathrm{E}(Y \mid L = 1, A = a) \cdot Pr(L = 1) \\ &\qquad + \; \mathrm{E}(Y \mid L = 0, A = a) \cdot Pr(L = 0) \end{aligned}$$

- More generally, for each a

$$\mathrm{E}(Y^a) = \sum_{\ell} \mathrm{E}(Y \mid L = \ell, A = a) \cdot Pr(L = \ell)$$

Standardization

$$E(Y^a) = \sum_{\ell} E(Y \mid L = \ell, A = a) Pr(L = \ell)$$



$$E(Y^a) = 0.4 \cdot \frac{1}{4} + 0.3 \cdot \frac{1}{4} + 0.5 \cdot \frac{1}{2}$$

Check Your Understanding: Standardization

$$E(Y^{a=0}) = \sum_\ell E(Y \mid L=\ell, A=0) Pr(L=\ell)$$

$$E(Y^{a=1}) = \sum_\ell E(Y \mid L=\ell, A=1) Pr(L=\ell)$$

		L	A	Y
1	Rheia	0	0	0
2	Kronos	0	0	1
3	Demeter	0	0	0
4	Hades	0	0	0
5	Hestia	0	1	0
6	Poseidon	0	1	0
7	Hera	0	1	0
8	Zeus	0	1	1

		L	A	Y
9	Artemis	1	0	1
10	Apollo	1	0	1
11	Leto	1	0	0
12	Ares	1	1	1
13	Athena	1	1	1
14	Hephaestus	1	1	1
15	Aphrodite	1	1	1
16	Polyphemus	1	1	1
17	Persephone	1	1	1
18	Hermes	1	1	0
19	Hebe	1	1	0
20	Dionysus	1	1	0

Check Your Understanding: Standardization

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$$Pr(L=1) \cdot \mathrm{E}(Y \mid L=1, A=0) =$$
$$Pr(L=0) \cdot \mathrm{E}(Y \mid L=0, A=0) =$$

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Check Your Understanding: Standardization

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$$Pr(L=1)\cdot \mathrm{E}(Y \mid L=1, A=0) = \tfrac{12}{20}\cdot\tfrac{2}{3}$$
$$Pr(L=0)\cdot \mathrm{E}(Y \mid L=0, A=0) = \tfrac{8}{20}\cdot\tfrac{1}{4}$$
$$Pr(L=1)\cdot \mathrm{E}(Y \mid L=1, A=1) = \tfrac{12}{20}\cdot\tfrac{6}{9}$$
$$Pr(L=0)\cdot \mathrm{E}(Y \mid L=0, A=1) = \tfrac{8}{20}\cdot\tfrac{1}{4}$$

Get $E(Y^{a=0})$ and $E(Y^{a=1})$ using formulas in slide 13!

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