Exchangeability continued

INFO/STSCI/ILRST 3900: Causal Inference

5 Sept 2024

Learning goals for today

At the end of class, you will be able to:

1. Explain what exchangeability is and why it is important
2. Explain the advantages and limitations of randomized experiments
3. Explain marginal versus conditional exchangeability

Logistics

- Problem Set 1 due Tuesday, Sept 10th at 5pm on Canvas
- Reading: Chapter 2 (through 2.3) of Hernán & Robins
- Post questions on Ed Discussion or come to office hours!
  - Filippo: **Mon** 11am-12pm (Comstock 1187)
  - Sam: **Tue** 4-5pm (Comstock 1187)
  - Shira: **Wed** 3-4pm (Comstock 1187)
  - Mayleen: **Thu** 10:15-11:15am (Rhodes 657, Room 2)

Check Your understanding: Exchangeability

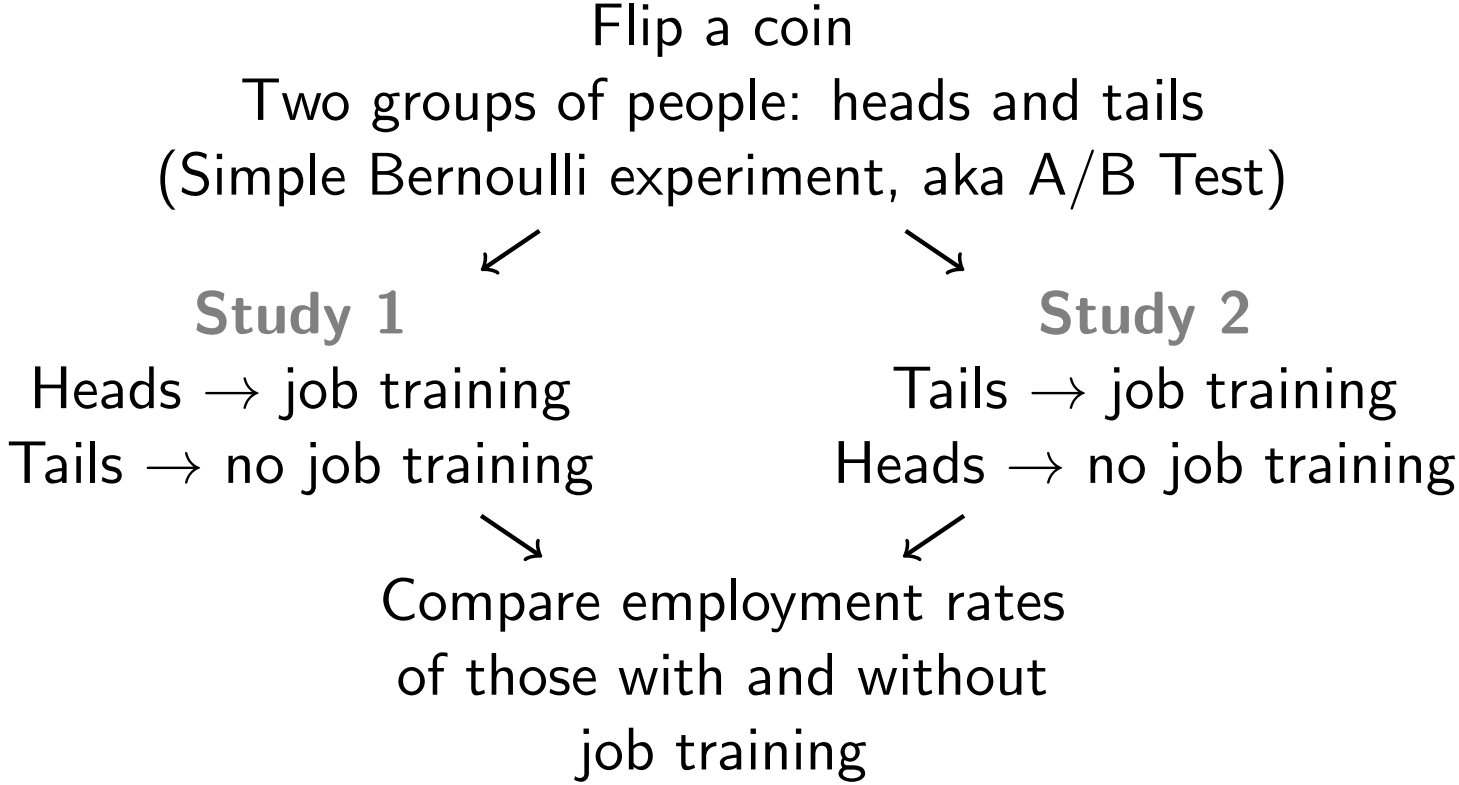
Definition: Exchangeability means that the potential outcomes, $Y^{a=1}$ and $Y^{a=0}$, are independent of the observed treatment (A):

$$\underbrace{Y^{a=1}, Y^{a=0}}_{\text{potential outcomes}} \perp\!\!\!\perp \underbrace{A}_{\text{observed treatment}}$$

In groups of 2-4, discuss the following:

1. (*everyone*) How is this different than $Y \perp\!\!\!\perp A$?
2. (*front half*) In randomized experiments, $Y^a \perp\!\!\!\perp A$ is usually true. Is $Y \perp\!\!\!\perp A$ ever true?
3. (*back half*) What would be an example of an experiment, random or not random, where exchangeability **does not** hold?

Exchangeability: By an experimental procedure



Question: Are both studies valid?

Yes. The (H/T) groups are **exchangeable**.
Any statistical pattern between (H/T) and employment
can only arise from the causal effect of job training

Why is exchangeability good?

Consider the average treatement effect (ATE):

$$\text{ATE} = \underbrace{\mathrm{E}(Y^{a=1})}_{\text{if everyone is treated}} - \underbrace{\mathrm{E}(Y^{a=0})}_{\text{if no-one is treated}}$$

Fundamental Problem of Causal Inference: missing data!

When exchangeability is true, it implies

$$\underbrace{\mathrm{E}(Y^{a=1} \mid A = 1)}_{\text{Within treated}} = \underbrace{\mathrm{E}(Y^{a=1} \mid A = 0)}_{\text{Within not treated}} = \underbrace{\mathrm{E}(Y^{a=1})}_{\text{everyone}}$$

This allows us to *identify* the ATE because we can plug-in

$$\underbrace{\mathrm{E}(Y^{a=1} \mid A = 1)}_{\substack{\text{outcomes for people who} \\ \text{are } \textbf{actually} \text{ treated}}} \quad \text{and} \quad \underbrace{\mathrm{E}(Y^{a=0} \mid A = 0)}_{\substack{\text{outcomes for people who} \\ \text{are } \textbf{actually} \text{ not treated}}}$$

Exchangeability lets us link a causal effect to observable data.

When does exchangeability hold?

- Data does not tell us directly whether exchangeability holds
- We must know how the data was gathered
- In a simple Bernoulli(p) randomized experiment (aka A/B Test, aka "flipping coins"), exchangeability holds by design
  - To estimate the causal effect from experimental data, we can simply take the difference in observed means
- When we have observational data, things get trickier

Check Your Understanding: Exchangeability ELI5 Edition

In the same groups as before, discuss the following:

- ELI5: How would you explain exchangeability and its importance to a five year old?
- Any lingering questions related to the concepts of exchangeability and randomized experiments

A volunteer from each group should be ready to give their "ELI5" explanation of exchangeability or ask a question about exchangeability your group still has.

Limits of randomized experiments

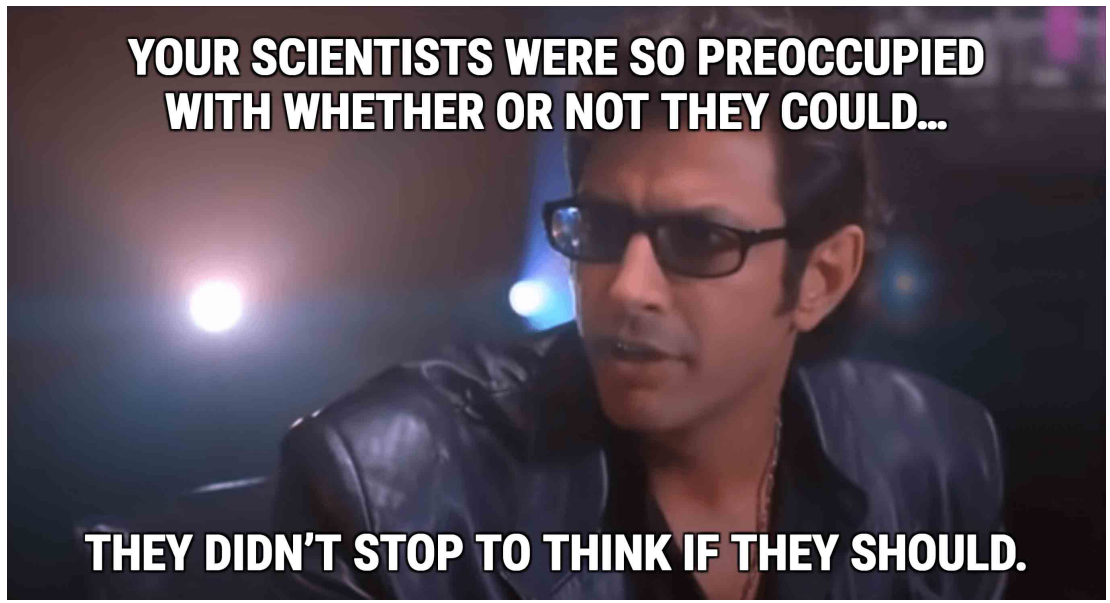
Randomized experiments are powerful tools for learning causal relationships, but they may not be possible because of

- **Feasibility**: What is the causal effect of capitalism on a country's economic outcomes?
- **Cost**: What is the causal effect of giving every student a Lamborghini on traffic in Collegetown?
- **Ethics**: What is the causal effect on cancer of smoking cigarettes?

Causal inference with observational data is very important!

Ethical Considerations

- Experiments may have negative effect on participants or larger population[1]
- Belmont Report[2]: Ethical principles and guidelines for the protection of human subjects of research



[1] Mcdermott and Hatemi PNAS 2020 https://www.pnas.org/doi/10.1073/pnas.2012021117

[2] https://www.hhs.gov/ohrp/regulations-and-policy/belmont-report/read-the-belmont-report/index.html

Conditional randomization

We like randomized experiments because they easily satisfy exchangeability, thus linking causal effects to observable data

Does exchangeability hold in every randomized experiment?

- Causal effect of job training on employment rate
- Suppose the researchers instead split the group into two groups
  - Age ≥ 18: Job training with probability $2/3$
  - Age < 18: Job training with probability $1/2$
- Does exchangeability still hold? Why or why not?

Conditional randomization

Exchangeability may not hold in every randomized experiment

- Age ≥ 18: more likely to get job training, more likely to have a job if treated
- Age < 18: less likely to get job training, less likely to have a job if treated
- Individuals who got job training ($A_i = 1$) are more likely to be employed ($Y_i^{a=1} = 1$) than individuals who didn't ($Y_i^{a=1} = 0$)
- In other words, the two groups are **not exchangeable**
- However, exchangeability holds *within* each sub-population
- Two separate experiments; both satisfy exchangeability

Check Your Understanding: Conditional randomization

- **Marginal exchangeability:** $Y^a \perp\!\!\!\perp A$ for all a
- **Conditional exchangeability:** $Y^a \perp\!\!\!\perp A \mid L$ for all a

  The potential outcomes are independent of treatment **conditional on** L (e.g. L could be an indicator of age group)

- Suppose you are interested in the causal effect of a COVID vaccine on whether or not a person contracts COVID
- How would you design a study such that marginal exchangeability holds? Conditional exchangeability?

Marginal or Conditional Exchangeability as a goal

- Random experiments are the "gold standard" for estimating causal effects because its easy to satisfy exchangeability
- Imagine the "ideal experiment" to answer your question; try to replicate the "ideal experiment" with observational analysis
- **When (marginal or conditional) exchangeability holds, it makes estimating causal effects from observable quantities possible**

- Why conditional randomization experiment?
  - Marginal exchangeability is very unlikely in observational data
  - Conditional exchangeability may be more reasonable

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At the end of class, you will be able to:

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After today's lecture...

- Get started on Problem Set 1 if you haven't yet!
- Read Chapter 2 (through 2.3) of Hernán & Robins
- Set up a PollEverywhere account **with your Cornell email**
- Come to next lecture with a question about the reading