Randomized experiments

INFO/STSCI/ILRST 3900: Causal Inference

3 Sep 2024

Learning goals for today

At the end of class, you will be able to:

1. Explain why exchangeability holds in randomized experiments
2. Understand why exchangeability allows for direct estimation of causal effects

Logistics

- Problem Set 1 will be released today

Potential outcome notation review

- We typically use i to denote a generic unit in our study
- Y_i is the observed outcome for unit i
- A_i is treatment received by unit i

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  - We typically use a to denote a generic treatment in our study
  - The **Consistency assumption** mean that the outcome we observe corresponds to the potential outcome of the observed treatment
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  - We typically use a to denote a generic treatment in our study
  - The **Consistency assumption** mean that the outcome we observe corresponds to the potential outcome of the observed treatment
  - We will refer to the unobserved potential outcome as the **counterfactual**
- The causal effect for individual i can be defined as

$$Y_i^{\text{treatment}} - Y_i^{\text{NoTreatment}}$$

Potential outcome exercise

- We observe that Martha ate a Mediterranean diet, and we observe that Martha survived.
  Suppose Martha had eaten a standard diet, we would have observed that Martha survived.
- We observe that Ezra ate a standard diet, and we observe that Ezra did not survive
  Suppose Ezra had eaten a Mediterranean diet, we would have observed that Ezra survived.

Potential outcome exercise

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  Suppose Martha had eaten a standard diet, we would have observed that Martha survived.
- We observe that Ezra ate a standard diet, and we observe that Ezra did not survive
  Suppose Ezra had eaten a Mediterranean diet, we would have observed that Ezra survived.

Assuming *consistency*, what is A_i, Y_i, $Y_i^{\texttt{MedDiet}}$ and $Y_i^{\texttt{StanDiet}}$,

- When $i =$ Martha?
- When $i =$ Ezra?

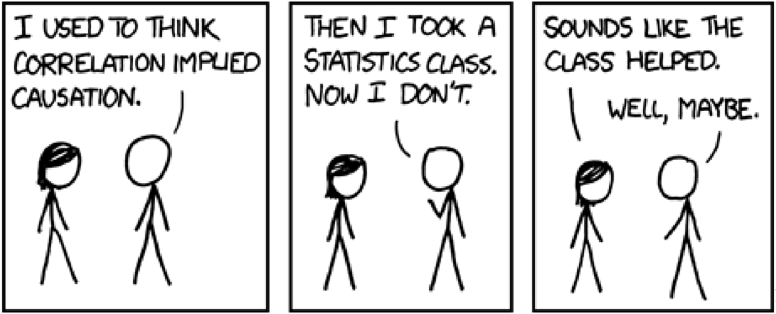
Potential outcome exercise

Suppose we know the following pieces of information:

- We observe that Martha ate a Mediterranean diet, and we observe that Martha survived.
  Suppose Martha had eaten a standard diet, we would have observed that Martha survived.
- We observe that Ezra ate a standard diet, and we observe that Ezra did not survive
  Suppose Ezra had eaten a Mediterranean diet, we would have observed that Ezra survived.

	A_i	Y_i	Y_i^{MedDiet}	Y_i^{StanDiet}
Martha				
Ezra				

Randomized experiments



[1]

[1] https://xkcd.com/552/

Randomized experiments

The New York Times

TheUpshot

What Coronavirus Researchers Can Learn From Economists

Randomized controlled trials remain the gold standard, but natural experiments can help doctors who need answers now.

Share full article 73

By Anupam B. Jena and Christopher M. Worsham

June 30, 2020

Fundamental problem of causal inference

- Randomized experiments are the gold standard for estimating causal effects
- Fundamental problem of causal inference is that we don't observe counterfactual outcomes
- Data is still missing in random experiments

	A	$Y^{a=1}$	$Y^{a=0}$	$Y^{a=1} - Y^{a=0}$
Ind 1	0	?	0	?
Ind 2	0	?	1	?
Ind 3	0	?	0	?
Ind 4	1	1	?	?
Ind 5	1	0	?	?
Ind 6	1	1	?	?

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- Why do randomized experiments "work"?

What can go wrong?

Suppose we tested all individuals in Ithaca in the Fall of 2021

- Whether the individual was vaccinated for Covid
  $A_i = 1$ if vaccinated, $A_i = 0$ if not vaccinated
- Whether the individual tested positive for Covid in 2021
  $Y_i = 1$ if positive test, $Y_i = 0$ if no positive test

What can go wrong?

Front of class

- Of the **vaccinated** individuals ($A_i = 1$), 50% had a positive Covid test ($Y_i = 1$)
$$\mathrm{E}(Y \mid A = 1) = .5$$
- Of the **not vaccinated** individuals ($A_i = 0$), 70% had a positive Covid test ($Y_i = 1$)
$$\mathrm{E}(Y \mid A = 0) = .7$$
- How might a vaccine skeptic explain the data?

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Back of class

- Of the individuals who are **vaccinated** ($A_i = 1$), 70% had a positive Covid test ($Y_i = 1$)

$$\mathrm{E}(Y \mid A = 1) = .7$$

- Of the individuals who are **not vaccinated** ($A_i = 0$), 50% had a positive Covid test ($Y_i = 1$)

$$\mathrm{E}(Y \mid A = 0) = .5$$

- How might a vaccine advocate explain the data?

Randomized experiment

Experiment designed by Pfizer **randomly assign** each individual (43,000 total) into two groups[2]:

- Two doses of BNT162b2 vaccine 21 days apart
- Two doses of placebo 21 days apart
- Whether a positive Covid test was recorded between 7 days and 14 weeks after the injection

[2] Polack et. al. NEJM 2020

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- Of the individuals who were given the vaccine ($A_i = 1$), 0.04% had a positive Covid test ($Y_i = 1$)
- Of the individuals who were given the placebo ($A_i = 0$), 0.9% had a positive Covid test ($Y_i = 1$)
- Individuals who received the placebo were ≈ 20 times more likely to get Covid

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- Of the individuals who were given the placebo ($A_i = 0$), 0.9% had a positive Covid test ($Y_i = 1$)
- Individuals who received the placebo were $\approx$ 20 times more likely to get Covid

Do the skeptics objections still hold?

[2] Polack et. al. NEJM 2020

Why experiments "work"

Table 1. Demographic Characteristics of the Participants in the Main Safety Population.*			
Characteristic	**BNT162b2 (N=18,860)**	**Placebo (N=18,846)**	**Total (N=37,706)**
Sex — no. (%)			
Male	9,639 (51.1)	9,436 (50.1)	19,075 (50.6)
Female	9,221 (48.9)	9,410 (49.9)	18,631 (49.4)
Race or ethnic group — no. (%)†			
White	15,636 (82.9)	15,630 (82.9)	31,266 (82.9)
Black or African American	1,729 (9.2)	1,763 (9.4)	3,492 (9.3)
Asian	801 (4.2)	807 (4.3)	1,608 (4.3)
Native American or Alaska Native	102 (0.5)	99 (0.5)	201 (0.5)
Native Hawaiian or other Pacific Islander	50 (0.3)	26 (0.1)	76 (0.2)
Multiracial	449 (2.4)	406 (2.2)	855 (2.3)
Not reported	93 (0.5)	115 (0.6)	208 (0.6)
Hispanic or Latinx	5,266 (27.9)	5,277 (28.0)	10,543 (28.0)
Country — no. (%)			
Argentina	2,883 (15.3)	2,881 (15.3)	5,764 (15.3)
Brazil	1,145 (6.1)	1,139 (6.0)	2,284 (6.1)
South Africa	372 (2.0)	372 (2.0)	744 (2.0)
United States	14,460 (76.7)	14,454 (76.7)	28,914 (76.7)
Age group — no. (%)			
16–55 yr	10,889 (57.7)	10,896 (57.8)	21,785 (57.8)
>55 yr	7,971 (42.3)	7,950 (42.2)	15,921 (42.2)
Age at vaccination — yr			
Median	52.0	52.0	52.0
Range	16–89	16–91	16–91
Body-mass index‡			
≥30.0: obese	6,556 (34.8)	6,662 (35.3)	13,218 (35.1)

* Percentages may not total 100 because of rounding.
† Race or ethnic group was reported by the participants.
‡ The body-mass index is the weight in kilograms divided by the square of the height in meters.

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In random experiments, the distribution of **potential outcomes** for those who are treated and those who are not treated (control group) are similar!

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This is a concept called **exchangeability**

Sometimes also referred to as **exogenous** $\underbrace{\textit{exo}}_{\text{outside}} \cdot \underbrace{\textit{genous}}_{\text{to produce}}$

Exchangeability

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- Two random variables are **independent** if the outcome of one does not give any information about the outcome of the other
- What we would have observed if an individual was given the treatment ($Y^{a=1}$) is independent of whether or not the individual actually received treatment
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- Exchangeability means that the **potential** outcomes Y^a are independent of the observed treatment
- Exchangeability does **not** mean that the **observed** outcome Y is independent of the observed treatment!

Exchangeability

$A = 1$ means vaccinated; $A = 0$ means unvaccinated
$Y = 1$ means covid; $Y = 0$ means no covid;

	$Y^{a=1}$	$Y^{a=0}$	A	Y
Low Risk 1	0	0	?	?
Low Risk 2	0	0	?	?
High Risk 3	0	1	?	?
High Risk 4	0	1	?	?

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$$\text{Average Causal Effect} = \underbrace{\mathrm{E}(Y^{a=1})}_{0} - \underbrace{\mathrm{E}(Y^{a=0})}_{1/2} = -1/2$$

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Suppose Low Risk choose $A_i = 0$ and High Risk choose $A_i = 1$ so the potential outcomes are not independent of the observed treatment

$$\underbrace{\mathrm{E}(Y \mid A = 1)}_{\text{observed vax}} - \underbrace{\mathrm{E}(Y \mid A = 0)}_{\text{observed unvax}} = 0$$

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Exchangeability

In mathematical notation, exchangeability means

$$\underbrace{Y^{a=1}, Y^{a=0}}_{\text{potential outcomes}} \perp\!\!\!\perp \underbrace{A}_{\text{observed treatment}}$$

Why is exchangeability good?

The average causal effect (ACE) is the difference in average outcome that would occur if everyone is treated compared to the average outcome that would occur if no-one is treated

$$\text{ACE} = \text{E}(Y^{a=1} - Y^{a=0}) = \underbrace{\text{E}(Y^{a=1})}_{\text{if everyone is treated}} - \underbrace{\text{E}(Y^{a=0})}_{\text{if no-one is treated}}$$

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The problem is, we only know

- $\text{E}(Y^{a=1} \mid A = 1)$, the average $Y^{a=1}$ among individuals who are actually treated
- $\text{E}(Y^{a=0} \mid A = 0)$, the average $Y^{a=0}$ among individuals who are actually not treated

Why is exchangeability good?

When exchangeability is true, it implies

$$\underbrace{\mathrm{E}(Y^{a=1} \mid A = 1)}_{\text{Within treated}} = \underbrace{\mathrm{E}(Y^{a=1} \mid A = 0)}_{\text{Within not treated}} = \underbrace{\mathrm{E}(Y^{a=1})}_{\text{everyone}}$$

Why is exchangeability good?

When exchangeability is true, it implies

$$\underbrace{\mathrm{E}(Y^{a=1} \mid A=1)}_{\text{Within treated}} = \underbrace{\mathrm{E}(Y^{a=1} \mid A=0)}_{\text{Within not treated}} = \underbrace{\mathrm{E}(Y^{a=1})}_{\text{everyone}}$$

This allows us to identify the average causal effect (ACE)

$$\mathrm{ATE} = \underbrace{\mathrm{E}(Y^{a=1})}_{\text{if everyone is treated}} - \underbrace{\mathrm{E}(Y^{a=0})}_{\text{if no-one is treated}}$$

because we can plug-in

$$\underbrace{\mathrm{E}(Y^{a=1} \mid A=1)}_{\substack{\text{outcomes for people who} \\ \text{are } \textbf{actually} \text{ treated}}} \quad \text{and} \quad \underbrace{\mathrm{E}(Y^{a=0} \mid A=0)}_{\substack{\text{outcomes for people who} \\ \text{are } \textbf{actually} \text{ not treated}}}$$

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1. Explain why exchangeability holds in randomized experiments
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