Instrumental Variables

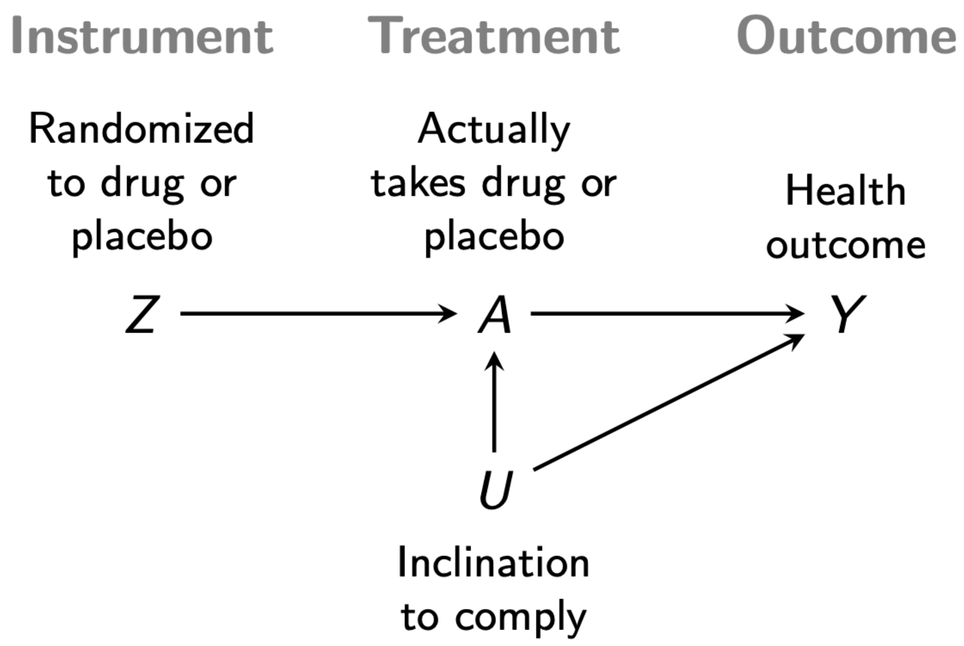
Discussion 9

Reminders and Announcements

- Problem Set 4 due Monday, **October 27**
- Office hours:
  - Filippo: Thursday 4-5 pm in 321A CIS Building
  - Shira: Monday 5-6 pm in 329A CIS Building
  - Sam: Tuesday 4-5 pm, in 350 CIS Building

Instrumental Variables

- What is the key assumption of IV?
- What is the intent to treat effect?
- What is the local average treatment effect?



Proportion of compliers

$$
\begin{aligned}
&\mathrm{E}(A \mid Z = 1) - \mathrm{E}(A \mid Z = 0) = \mathrm{E}(A^{Z=1} - A^{Z=0}) \\
&= \sum_s \mathrm{E}(A^{Z=1} - A^{Z=0} \mid S = s) \underbrace{\mathrm{P}(S = s)}_{\substack{\text{Denote} \\ \pi_s}} \\
&= \mathrm{E}(A^{Z=1} - A^{Z=0} \mid S = \text{Complier})\pi_{\text{Complier}} \\
&\qquad + \mathrm{E}(A^{Z=1} - A^{Z=0} \mid S = \text{Always-Taker})\pi_{\text{Always-Taker}} \qquad (= 0) \\
&\qquad + \mathrm{E}(A^{Z=1} - A^{Z=0} \mid S = \text{Never-Taker})\pi_{\text{Never-Taker}} \qquad (= 0) \\
&\qquad + \mathrm{E}(A^{Z=1} - A^{Z=0} \mid S = \text{Defier})\pi_{\text{Defier}} \qquad (= 0) \\
&= \pi_{\text{Complier}}
\end{aligned}
$$

ACE for Compliers

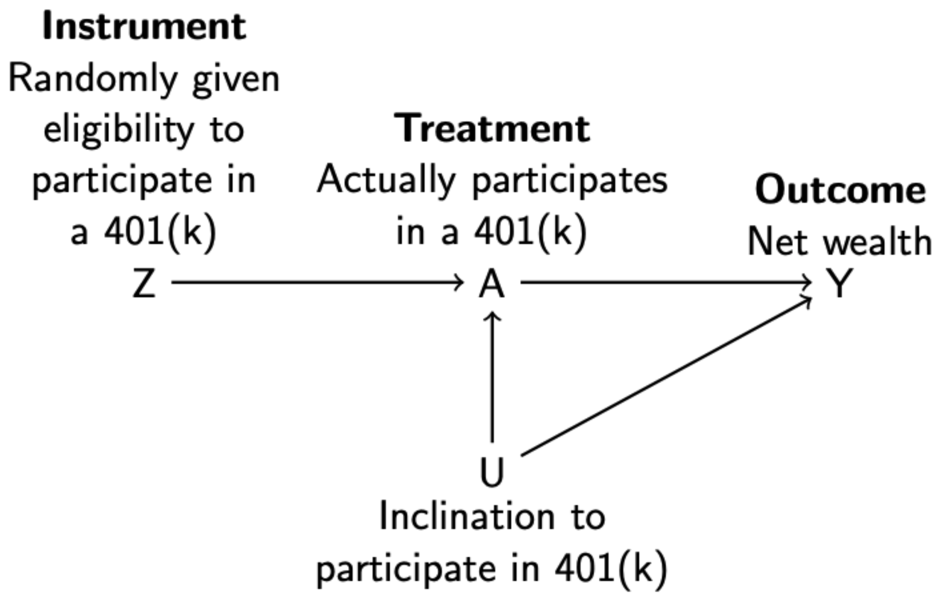
Deriving the general case:

$$
\begin{aligned}
\mathrm{E}(Y \mid Z=1) - \mathrm{E}(Y \mid Z=0) &= \mathrm{E}(Y^{Z=1} - Y^{Z=0}) \\
&= \sum_s \mathrm{E}(Y^{Z=1} - Y^{Z=0} \mid S = s) \underbrace{\mathrm{P}(S = s)}_{\substack{\text{Denote} \\ \pi_s}} \\
&= \mathrm{E}(Y^{Z=1} - Y^{Z=0} \mid S = \text{Complier}) \pi_{\text{Complier}} \\
&\quad + \mathrm{E}(Y^{Z=1} - Y^{Z=0} \mid S = \text{Always-Taker}) \pi_{\text{Always-Taker}} \qquad (= 0) \\
&\quad + \mathrm{E}(Y^{Z=1} - Y^{Z=0} \mid S = \text{Never-Taker}) \pi_{\text{Never-Taker}} \qquad (= 0) \\
&\quad + \mathrm{E}(Y^{Z=1} - Y^{Z=0} \mid S = \text{Defier}) \pi_{\text{Defier}} \qquad (= 0)
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{E}(Y^{A=1} - Y^{A=0} \mid S = \text{Complier}) &= \frac{\mathrm{E}(Y^{Z=1} - Y^{Z=0})}{\pi_{\text{Complier}}} \\
&= \frac{\mathrm{E}(Y \mid Z=1) - \mathrm{E}(Y \mid Z=0)}{\mathrm{E}(A \mid Z=1) - \mathrm{E}(A \mid Z=0)}
\end{aligned}
$$

401(k) Example

- Does participating in a 401(k) increase an individual’s wealth?
- Participating in a 401(k) is not a random thing!
- However, being eligible for a 401(k) is arguably random.
- 401(k) eligibility affects net wealth *only* through participation.



401(k) Example

- Describe what the intent to treat effect is?
- Describe who are the always-takers? Never-takers? Compliers?
- What would it look like in this context if someone was a defier?
- Why does it matter that our instrument (Z) is assigned randomly? (In other words, what assumption becomes credible because (Z) is random?)

Estimation (Two-stage least squares)

- Since Z is assigned randomly, the intent to treat effect is β_i in the following regression:
$$wealth_i = \alpha_i + \beta_i * eligibility + \epsilon_i$$
- The average effect among compliers isn't quite so simple.
  - First estimate the treatment as a function of the instrument
$$treatment_i = \alpha_i + \beta_i * eligibility_i + \epsilon_i$$
  - Then replace binary treatment (0, 1, 1, 0, …) with predicted probabilities from this model (0.2, 0.8, 0.9, 0.1, …)
  - Finally, estimate the regression:
$$wealth_i = \alpha_i + \beta_i * probability_i + \epsilon_i. \beta_i$$
is the average effect among compliers.
- We usually use canned software for this… e.g. in R!

Let’s do it ourselves!

- There is a short coding exercise on the website…