IPW and AIPW Lab

INFO/STSCI/ILRST 3900: Causal Inference

4 Nov 2026

IPW Review

- Each individual should be reweighted by

$$\text{Weight} = \frac{1}{\text{Prob of their obs treatment status given their L}}$$

- Propensity Score: $\pi_i = Pr(A_i = 1 \mid L_i)$
- Model the treatment assignment

$$\hat{\pi}_i = \hat{\mathrm{P}}(A = 1 \mid L)$$

- **Estimate** by inverse probability weighting (IPW)

$$\begin{aligned}\widehat{\mathrm{ATE}} &= \hat{\mathrm{E}}(Y^1) - \hat{\mathrm{E}}(Y^0) \\ &= \underbrace{\frac{1}{N}\sum_{i:A_i=1}\frac{1}{\hat{\pi}_i} \times Y_i}_{\text{Weighted avg of treated obs}} - \underbrace{\frac{1}{N}\sum_{i:A_i=0}\frac{1}{1-\hat{\pi}_i} \times Y_i}_{\text{Weighted Avg of untreated obs}}\end{aligned}$$

Augmented IPW Review

- G-formula: learn a model to predict Y given L and A

$$\hat{\mathrm{E}}(Y \mid A = 0, L) \quad \text{and} \quad \hat{\mathrm{E}}(Y \mid A = 1, L)$$

- AIPW uses both estimates for outcome and propensity scores

$$\hat{E}(Y^{a=1}) = \frac{1}{N}\left(\underbrace{\sum_i \hat{E}(Y \mid A = 1, L = \ell_i)}_{\text{g-computation}} + \underbrace{\sum_i \frac{A_i Y_i}{\hat{\pi}_i}}_{\text{IPW}} - \underbrace{\sum_i \frac{A_i}{\hat{\pi}_i}\hat{E}(Y \mid A = 1, L = \ell_i)}_{\text{new term}}\right)$$

$$\hat{E}(Y^{a=0}) = \frac{1}{N}\left(\underbrace{\sum_i \hat{E}(Y \mid A = 0, L = \ell_i)}_{\text{g-computation}} + \underbrace{\sum_i \frac{(1 - A_i) Y_i}{1 - \hat{\pi}_i}}_{\text{IPW}} - \underbrace{\sum_i \frac{1 - A_i}{1 - \hat{\pi}_i}\hat{E}(Y \mid A = 0, L = \ell_i)}_{\text{new term}}\right)$$

- Enough that the predicted outcome or propensity score is correct:

$$\hat{E}(Y \mid A, L = \ell_i) = E(Y \mid A, L = \ell_i) \quad \text{or} \quad \hat{\pi} = \pi$$

- Double robustness:
  robust against misspecification of either (but not both)