

Exchangeability and Consistency

STSCI/INFO/ILRST 3900: Causal Inference

September 18, 2024

Agenda

- Reminders and Announcements
- Class activity
- Homework Check-in and Questions

Reminders and Announcements

- HW 2 due Tuesday (September 24) by 5pm
- Submit the PDF file

The image shows a screenshot of the RStudio interface. On the left, the R Markdown source file is open, showing code for generating a PDF. A green arrow points from the 'Knit to PDF' option in the 'Knit' menu to the rendered PDF output on the right. The code in the source file is as follows:

```
1- example.Rmd
1 - 
2 t 
3 o 
4 - 
5 
6 { include = FALSE }
7 library(viridis)
8 ```
9 
10 The code below demonstrates two color palettes in the
11 [viridis](https://github.com/sjmgarnier/viridis) package.
12 Each plot displays a contour map of the Maunga Whau volcano
13 in Auckland, New Zealand.
14 
15 ## Viridis colors
16 ```{r}
17 image(volcano, col = viridis(200))
18 ```
19 
20 ## Magma colors
21 ```{r}
22 image(volcano, col = viridis(200, option = "A"))
23 ```
```

The rendered PDF on the right, titled 'Viridis Demo', contains the following text and plots:

Viridis Demo

The code below demonstrates two color palettes in the viridis package. Each plot displays a contour map of the Maunga Whau volcano in Auckland, New Zealand.

Viridis colors

```
image(volcano, col = viridis(200))
```

Magma colors

```
image(volcano, col = viridis(200, option = "A"))
```

Two contour maps of the Maunga Whau volcano are shown, one using the viridis color palette and the other using the magma color palette. The viridis map shows a yellow and green center, while the magma map shows a red and orange center.

POINTS TAKEN OFF FOR INCORRECT FORMATTING

Formatted correctly, NO points taken off for formatting

Reminders and Announcements

- Office Hours
 - **Filippo:** Monday 11am-12pm in Comstock 1187
 - **Shira:** Wednesday 5:30-6:30pm in in Comstock 1187
 - See Ed Discussion for Zoom links/info

Consistency and Exchangeability

- **Consistency**- the observed outcome equals the potential outcome that corresponds to the given treatment

$$Y_i = \begin{cases} Y_i^{a=1} & A_i = 1 \\ Y_i^{a=0} & A_i = 0 \end{cases}$$

- **Exchangeability**- the treatment assignment is independent of the potential outcomes

$$P(Y^a = 1 | A = 1) = P(Y^a = 1 | A = 0)$$

Class Activity

1. Fill in the blanks such that **exchangeability** holds
2. Fill in the observed outcome (Y) such that **consistency** holds
3. Calculate the quantities at the bottom (without simplifying)

Note:

$$E[Y^{a=1}] \stackrel{\text{exchangeability}}{=} E[Y^{a=1} | A = 1] \stackrel{\text{consistency}}{=} E[Y | A = 1]$$

	i	$Y^{a=0}$	$Y^{a=1}$	A	Y
1	Rheia	0	1	0	
2	Kronos	1	0	1	
3	Demeter	0	0	0	
4	Hades	1	0	0	
5	Hestia	0	0	1	
6	Poseidon	1	0		

For example: $E[Y^{a=0}] = 1 \times P(Y^{a=0} = 1) + 0 \times P(Y^{a=0} = 0) = \frac{3}{6}$

$$E[Y^{a=0} | A = 0] = \dots = \begin{cases} \frac{1}{3} & \text{if } A_6 = 1 \\ \frac{1+1}{3+1} & \text{if } A_6 = 0 \end{cases}$$

* This data is slightly different than the one in the assignment where $N = 20$

Class Activity

Quantities to calculate

- $E[Y^{a=0}]$ - the expected value of the **potential** outcome under no treatment
- $E[Y^{a=0} | A = 0]$ - the expected value of the **potential** outcome under no treatment conditioned on the control group (i.e. the expected value of the **potential** outcome under no treatment within the control group)
- $E[Y | A = 0]$ - the expected value of the **observed** outcome conditioned on the control group (i.e. the expected value of the **observed** outcome within the control group)

Class Activity

Step 1

	i	$Y^{a=0}$	$Y^{a=1}$	A	Y
1	Rhea	0	1	0	
2	Kronos	1	0	1	
3	Demeter	0	0	0	
4	Hades	0	0	0	
5	Hestia	0	0	1	
6	Poseidon	1	0	0	
7	Hera	0	0	1	
8	Zeus	0	1	1	
9	Artemis	1	1	0	
10	Apollo	1	0	0	

11	Leto	0	1		
12	Ares	1	1		
13	Athena	1	1	0	
14	Hephaestus	0	0	1	
15	Aphrodite	0	0	1	
16	Polyphemus	0	1	1	
17	Persephone	1	1	1	
18	Hermes	1	0	1	
19	Hebe	1	1	1	
20	Dionysus	1	1	1	

$$E[Y^{a=0}] = 1 \times P(Y^{a=0} = 1) + 0 \times P(Y^{a=0} = 0) = 1 \times \frac{10}{20} + 0 \times \frac{10}{20} = \frac{10}{20}$$

Class Activity

Step 2

	i	$Y^{a=0}$	$Y^{a=1}$	A	Y
1	Rheia	0	1	0	
2	Kronos	1	0	1	
3	Demeter	0	0	0	
4	Hades	0	0	0	
5	Hestia	0	0	1	
6	Poseidon	1	0	0	
7	Hera	0	0	1	
8	Zeus	0	1	1	
9	Artemis	1	1	0	
10	Apollo	1	0	0	

11	Leto	0	1		
12	Ares	1	1		
13	Athena	1	1	0	
14	Hephaestus	0	0	1	
15	Aphrodite	0	0	1	
16	Polyphemus	0	1	1	
17	Persephone	1	1	1	
18	Hermes	1	0	1	
19	Hebe	1	1	1	
20	Dionysus	1	1	1	

Ignoring the missing entries,

$$E[Y^{a=0} | A = 0] = \frac{P(Y^{a=0} = 1 \text{ and } A = 0)}{P(A = 0)} = \frac{4}{7}$$

Class Activity

Step 3

- For each value of A_{11} and A_{12} recalculate $E[Y^{a=0} | A = 0]$

$$E[Y^{a=0} | A = 0] = \begin{cases} \frac{4}{7} & \text{if } A_{11} = 1 \text{ and } A_{12} = 1 \\ \frac{4}{8} & \text{if } A_{11} = 0 \text{ and } A_{12} = 1 \\ \frac{5}{8} & \text{if } A_{11} = 1 \text{ and } A_{12} = 0 \\ \frac{5}{9} & \text{if } A_{11} = 0 \text{ and } A_{12} = 0 \end{cases}$$

- For exchangeability to hold we want $E[Y^{a=0}] = E[Y^{a=0} | A = 0]$ then:
 - $A_{11} = 0$ and $A_{12} = 1$

Class Activity

Step 4

	i	$Y^{a=0}$	$Y^{a=1}$	A	Y
1	Rheaia	0	1	0	0
2	Kronos	1	0	1	0
3	Demeter	0	0	0	
4	Hades	0	0	0	
5	Hestia	0	0	1	
6	Poseidon	1	0	0	
7	Hera	0	0	1	
8	Zeus	0	1	1	
9	Artemis	1	1	0	
10	Apollo	1	0	0	

11	Leto	0	1	0
12	Ares	1	1	1
13	Athena	1	1	0
14	Hephaestus	0	0	1
15	Aphrodite	0	0	1
16	Polyphemus	0	1	1
17	Persephone	1	1	1
18	Hermes	1	0	1
19	Hebe	1	1	1
20	Dionysus	1	1	1

Fill in the **observed** outcome such that:

$$Y_i = \begin{cases} Y_i^{a=1} & A_i = 1 \\ Y_i^{a=0} & A_i = 0 \end{cases}$$

For example: $A_1 = 0$ then $Y_1 = Y_1^{a=0} = 0$

$A_2 = 1$ then $Y_2 = Y_2^{a=1} = 0$

Class Activity

Step 5

	i	$Y^{a=0}$	$Y^{a=1}$	A	Y
1	Rhea	0	1	0	0
2	Kronos	1	0	1	0
3	Demeter	0	0	0	0
4	Hades	0	0	0	0
5	Hestia	0	0	1	0
6	Poseidon	1	0	0	1
7	Hera	0	0	1	0
8	Zeus	0	1	1	1
9	Artemis	1	1	0	1
10	Apollo	1	0	0	1

11	Leto	0	1	0	0
12	Ares	1	1	1	1
13	Athena	1	1	0	1
14	Hephaestus	0	0	1	0
15	Aphrodite	0	0	1	0
16	Polyphemus	0	1	1	1
17	Persephone	1	1	1	1
18	Hermes	1	0	1	0
19	Hebe	1	1	1	1
20	Dionysus	1	1	1	1

$$E[Y|A = 0] = \frac{0 + 0 + 0 + 1 + 1 + 1 + 0 + 1}{8} = \frac{4}{8}$$

Class Activity

- Repeat for $A = 1$ to obtain:

- $E[Y^{a=1}] = \frac{10}{20}$

- $E[Y^{a=1} | A = 1] = \frac{6}{12}$

- $E[Y | A = 1] = \frac{6}{12}$



Questions about the HW?