Exchangeability and Consistency

STSCI/INFO/ILRST 3900: Causal Inference

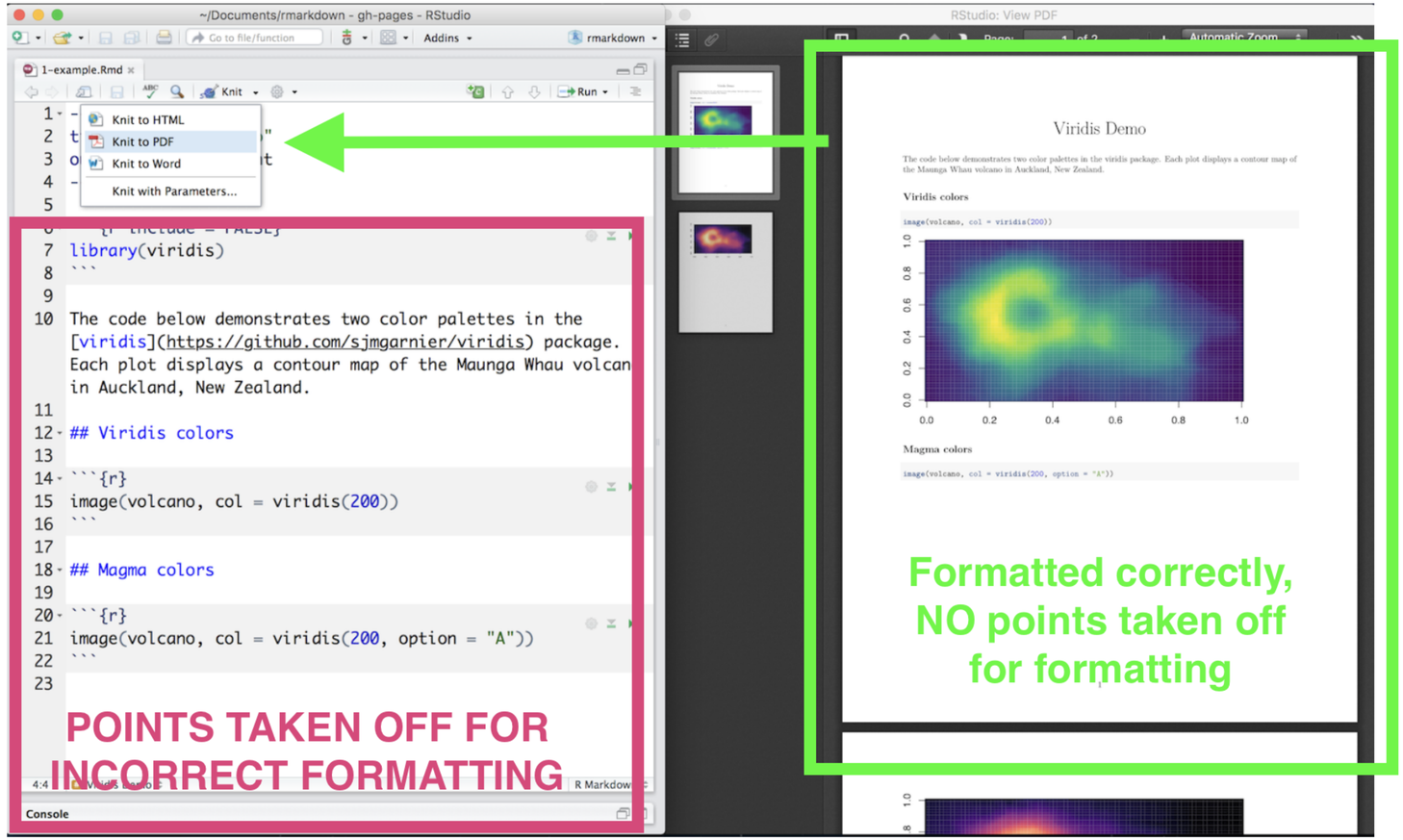
September 18, 2024

Agenda

- Reminders and Announcements
- Class activity
- Homework Check-in and Questions

Reminders and Announcements

- HW 2 due Tuesday (September 24) by 5pm
- Submit the PDF file



Reminders and Announcements

- Office Hours
  - **Filippo:** Monday 11am-12pm in Comstock 1187
  - **Shira:** Wednesday 5:30-6:30pm in in Comstock 1187
  - See Ed Discussion for Zoom links/info

Consistency and Exchangeability

- **Consistency-** the observed outcome equals the potential outcome that corresponds to the given treatment

$$Y_i = \begin{cases} Y_i^{a=1} & A_i = 1 \\ Y_i^{a=0} & A_i = 0 \end{cases}$$

- **Exchangeability**- the treatment assignment is independent of the potential outcomes

$$P(Y^a = 1 \,|\, A = 1) = P(Y^a = 1 \,|\, A = 0)$$

Class Activity

1. Fill in the blanks such that **exchangeability** holds
2. Fill in the observed outcome (Y) such that **consistency** holds
3. Calculate the quantities at the bottom (without simplifying)

Note:

$$E[Y^{a=1}] \overset{\text{exchangeability}}{=} E[Y^{a=1}|A=1] \overset{\text{consistency}}{=} E[Y|A=1]$$

	i	$Y^{a=0}$	$Y^{a=1}$	A	Y
1	Rheia	0	1	0	
2	Kronos	1	0	1	
3	Demeter	0	0	0	
4	Hades	1	0	0	
5	Hestia	0	0	1	
6	Poseidon	1	0		

For example: $E[Y^{a=0}] = 1 \times P(Y^{a=0}=1) + 0 \times P(Y^{a=0}=0) = \frac{3}{6}$

$$E[Y^{a=0}|A=0] = \ldots = \begin{cases} \frac{1}{3} & \text{if } A_6 = 1 \\ \frac{1+1}{3+1} & \text{if } A_6 = 0 \end{cases}$$

* This data is slightly different than the one in the assignment where $N = 20$

Class Activity

Quantities to calculate

- $E[Y^{a=0}]$ **-** the expected value of the **potential** outcome under no treatment
- $E[Y^{a=0}|A=0]$ **-** the expected value of the **potential** outcome under no treatment conditioned on the control group (i.e. the expected value of the **potential** outcome under no treatment within the control group)
- $E[Y|A=0]$ **-** the expected value of the **observed** outcome conditioned on the control group (i.e. the expected value of the **observed** outcome within the control group)

Class Activity

Step 1

	i	$Y^{a=0}$	$Y^{a=1}$	A	Y
1	Rheia	0	1	0	
2	Kronos	1	0	1	
3	Demeter	0	0	0	
4	Hades	0	0	0	
5	Hestia	0	0	1	
6	Poseidon	1	0	0	
7	Hera	0	0	1	
8	Zeus	0	1	1	
9	Artemis	1	1	0	
10	Apollo	1	0	0	
11	Leto	0	1		
12	Ares	1	1		
13	Athena	1	1	0	
14	Hephaestus	0	0	1	
15	Aphrodite	0	0	1	
16	Polyphemus	0	1	1	
17	Persephone	1	1	1	
18	Hermes	1	0	1	
19	Hebe	1	1	1	
20	Dionysus	1	1	1	

$$E[Y^{a=0}] = 1 \times P(Y^{a=0} = 1) + 0 \times P(Y^{a=0} = 0) = 1 \times \frac{10}{20} + 0 \times \frac{10}{20} = \frac{10}{20}$$

Class Activity

Step 2

	i	$Y^{a=0}$	$Y^{a=1}$	A	Y
1	Rheia	0	1	0	
2	Kronos	1	0	1	
3	Demeter	0	0	0	
4	Hades	0	0	0	
5	Hestia	0	0	1	
6	Poseidon	1	0	0	
7	Hera	0	0	1	
8	Zeus	0	1	1	
9	Artemis	1	1	0	
10	Apollo	1	0	0	
11	Leto	0	1		
12	Ares	1	1		
13	Athena	1	1	0	
14	Hephaestus	0	0	1	
15	Aphrodite	0	0	1	
16	Polyphemus	0	1	1	
17	Persephone	1	1	1	
18	Hermes	1	0	1	
19	Hebe	1	1	1	
20	Dionysus	1	1	1	

Ignoring the missing entries,

$$E[Y^{a=0}|A=0] = \frac{P(Y^{a=0}=1 \text{ and } A=0)}{P(A=0)} = \frac{4}{7}$$

Class Activity

Step 3

- For each value of A_{11} and A_{12} recalculate $E[Y^{a=0} | A = 0]$
- $$E[Y^{a=0} | A = 0] = \begin{cases} \frac{4}{7} & \text{if } A_{11} = 1 \text{ and } A_{12} = 1 \\ \frac{4}{8} & \text{if } A_{11} = 0 \text{ and } A_{12} = 1 \\ \frac{5}{8} & \text{if } A_{11} = 1 \text{ and } A_{12} = 0 \\ \frac{5}{9} & \text{if } A_{11} = 0 \text{ and } A_{12} = 0 \end{cases}$$
- For exchangeability to hold we want $E[Y^{a=0}] = E[Y^{a=0} | A = 0]$ then:
  - $A_{11} = 0$ and $A_{12} = 1$

Class Activity

Step 4

	i	$Y^{a=0}$	$Y^{a=1}$	A	Y
1	Rheia	0	1	0	0
2	Kronos	1	0	1	0
3	Demeter	0	0	0	
4	Hades	0	0	0	
5	Hestia	0	0	1	
6	Poseidon	1	0	0	
7	Hera	0	0	1	
8	Zeus	0	1	1	
9	Artemis	1	1	0	
10	Apollo	1	0	0	
11	Leto	0	1	0	
12	Ares	1	1	1	
13	Athena	1	1	0	
14	Hephaestus	0	0	1	
15	Aphrodite	0	0	1	
16	Polyphemus	0	1	1	
17	Persephone	1	1	1	
18	Hermes	1	0	1	
19	Hebe	1	1	1	
20	Dionysus	1	1	1	

Fill in the **observed** outcome such that:

$$Y_i = \begin{cases} Y_i^{a=1} & A_i = 1 \\ Y_i^{a=0} & A_i = 0 \end{cases}$$

For example: $A_1 = 0$ then $Y_1 = Y_1^{a=0} = 0$

$A_2 = 1$ then $Y_2 = Y_2^{a=1} = 0$

Class Activity

Step 5

	i	$Y^{a=0}$	$Y^{a=1}$	A	Y
1	Rheia	0	1	0	0
2	Kronos	1	0	1	0
3	Demeter	0	0	0	0
4	Hades	0	0	0	0
5	Hestia	0	0	1	0
6	Poseidon	1	0	0	1
7	Hera	0	0	1	0
8	Zeus	0	1	1	1
9	Artemis	1	1	0	1
10	Apollo	1	0	0	1
11	Leto	0	1	0	0
12	Ares	1	1	1	1
13	Athena	1	1	0	1
14	Hephaestus	0	0	1	0
15	Aphrodite	0	0	1	0
16	Polyphemus	0	1	1	1
17	Persephone	1	1	1	1
18	Hermes	1	0	1	0
19	Hebe	1	1	1	1
20	Dionysus	1	1	1	1

$$E[Y|A=0] = \frac{0+0+0+1+1+1+0+1}{8} = \frac{4}{8}$$

Class Activity

- Repeat for $A = 1$ to obtain:
  - $E[Y^{a=1}] = \frac{10}{20}$
  - $E[Y^{a=1}|A = 1] = \frac{6}{12}$
  - $E[Y|A = 1] = \frac{6}{12}$

Questions about the HW?